ECONOMICS 7380
PROBLEM SET #1

1. Consider a community of 2 citizens that is holding a referendum. Each citizen is either a supporter or an opposer of the referendum. Supporters are each willing to pay b for the proposed change, while opposers are each willing to pay x to avoid it. Each citizen knows whether he is a supporter or an opposer, but does not know the preferences of the other citizen. Both citizens know that the probability that a randomly selected individual is a supporter is μ. Citizens must decide whether to not to vote in the referendum. If they do vote, supporters vote in favor and opposers vote against. If the number of votes in favor of the referendum is at least as big as the number against, the proposed change is approved. Each citizen $i \in \{1, 2\}$ faces a cost of voting c_i where c_i is the realization of a random variable uniformly distributed on $[0, \overline{c}]$. Each citizen observes his own voting cost, but only knows that the cost of the other citizen is the realization of a random variable uniformly distributed on $[0, \overline{c}]$. Solve for the Bayesian Nash equilibrium of this game under the assumption that all supporters use the same strategy and all opposers use the same strategy.

2. Consider the example used in class to illustrate the Rule Utilitarian view of duty. Extend the example to have 3 citizens as opposed to 2. Keep everything else about the example the same, except assume that the reform will be approved only if at least 2 citizens vote in favor (as opposed to 1). Solve for the voting rule that a rule utilitarian would use.

3. Consider the model used in the section on Voting in Multicandidate Elections. Assume that $n = 7$ and that voters' utility functions are such that:

$$\begin{aligned}
V_{1A} &> V_{1B} > V_{1C} \\
V_{2A} &> V_{2B} > V_{2C} \\
V_{3B} &> V_{3A} > V_{3C} \\
V_{4B} &> V_{4A} > V_{4C} \\
V_{5C} &> V_{5B} > V_{5A} \\
V_{6C} &> V_{6B} > V_{6A} \\
V_{7C} &> V_{6B} > V_{6A}
\end{aligned}$$

Describe voting decisions that are consistent with strategic voting in which candidate A wins and in which candidate B wins. Which candidate would win with sincere voting?

4. Consider the following model of two candidate competition with policy-motivated candidates. Votes have quadratic preferences, so that the utility obtained by a voter of ideology i from electing candidate J is $-(i_J - i)^2$. Voters' ideologies are uniformly distributed over the interval $[0, 1]$, so that the fraction of voters with ideology $\leq i$ is just i. The two candidates have quadratic preferences with true ideologies $t_A = t$ and $t_B = 1 - t$ where $t < 1/2$. They also get a non-policy related reward to holding office $r > 0$ if they win. Consider the game in which the two candidates choose ideologies i_A and i_B and then voters vote for the candidate whose announced ideology they prefer. Demonstrate that if (i_A^*, i_B^*) is an equilibrium of this game, then $i_A^* = i_B^* = 1/2$.

5. Consider the following political agency model. There are two time periods, period 1 and period 2. There are two politicians - an incumbent and a challenger - and a representative voter. The incumbent holds office in period 1, but at the beginning of period 2 faces an election against the challenger to determine who holds office in period 2. In each period, the politician who holds office has to choose policy a or policy b. The voter prefers policy a. Each politician can be *congruent* or *dissonant.* Congruent politicians prefer policy a and dissonant politicians prefer policy b. When in office, each type of politician (i.e., congruent or dissonant) gets a policy-related payoff of 1 from choosing his preferred policy and 0 from the other policy. Each type of politican also gets a non-policy related payoff r when in office. When not in office, politicians get a payoff of 0. Politicians discount period 2 payoffs at rate $\delta < 1$. The voter cannot directly observe politicians' types. At the beginning of period 1, the voter believes the incumbent is congruent with probability π_I. The voter is assumed to observe the incumbent's period 1 policy choice and to update his beliefs about the incumbent's type rationally. The voter believes the challenger is congruent with probability π_C.

The game played between the politicians and the voter is as follows. In period 1, the incumbent chooses the period 1 policy a or b. At the beginning of period 2, the election is held and the voter chooses whether to re-elect the incumbent or elect the challenger. In period 2, the winning politician chooses the period 2 policy a or b. An *equilibrium* consists of (i) a strategy for the incumbent which specifies as a function of his type what policy he will choose in period 1 and, if re-elected, in period 2; (ii) a strategy for the challenger which specifies as a function of his type what policy he will choose if elected in period 2; (iii) beliefs for the voter which describe as a function of the incumbent's policy choice what he believes at the time of the election about the incumbent's type; and (iv) a strategy for the voter which describes as a function of the incumbent's policy choice whether he will re-elect the incumbent or elect the challenger. Each player's strategy has to be optimal given the other players' strategies and the voter's beliefs have to be consistent with the incumbent's strategy on the equilibrium path.

(a) Under what conditions does there exist a *separating equilibrium* in which the incumbent simply chooses his preferred policy in period 1?

(b) Under what conditions does there exist a *pooling equilibrium* in which both types of incumbent choose the voter's preferred policy in period 1?

PROBLEM SET #2

1. Consider a town school board consisting of three representatives. Representative 1 represents the rich area of town, Representative 2 the middle class area, and Representative 3 the poor area. The school board must choose how much to spend on the town's public school. There are three possible spending levels: high (p_h), medium (p_m), and low (p_l), where $p_h > p_m > p_l$. School spending is financed by taxes on the town's residents.

Representative 1's utility function is such that

$$V_1(p_h) > V_1(p_l) > V_1(p_m).$$

This reflects the fact that the rich will send their children to private school unless the public school is funded at a high level and, if they are not going to use the public school, they prefer p_l to p_m because their taxes will be lower. Representative 2's utility function is such that

$$V_2(p_m) > V_2(p_h) > V_2(p_l).$$

This reflects the fact that the middle class will always use the public school and see p_m as the optimal balance between school quality and affordable taxes. Representative 3's utility function is such that

$$V_3(p_l) > V_3(p_m) > V_3(p_h).$$

This reflects the fact that the poor will always use the public school and, having less income to pay taxes, see p_l as the optimal balance between school quality and affordable taxes.

(i) Show that there exists no Condorcet winner in this example.

(ii) Show that the representatives' utility functions do not satisfy the single-crossing property.

(iii) Suppose that the school board uses the following simple decision-making mechanism. One representative is randomly selected to propose a level of spending. The representatives then all vote for or against this proposal. If there are at least 2 votes for, the proposal is implemented. Otherwise, spending remains at the status quo level, which is p_m. Describe what will happen. How would your answer change if the status quo spending level were p_l?

2. **(a)** Consider the following special case of the common agency model discussed in class. There is a single interest group with policy preferences $V_1(p) = \alpha_1 \ln p - p$ and a politician with policy preferences $V_0(p) = \alpha_0 \ln p - p$. Solve for the equilibrium policy and contribution.

(b) In Example 2 presented in class, show that the equilibrium in which $p^* = 0$ is not a truthful equilibrium.

(c) Find a truthful equilibrium for Example 2.

3. Consider a legislature consisting of n legislators indexed by $i = 1, ..., n$. Assume n is odd and ≥ 3 and suppose that the legislature operates by majority rule. Suppose that the legislators are choosing the amount to be spent on district specific transfers. Thus, a policy is a vector $(p_1, ..., p_n)$ where p_i is the transfers to legislator i's district. There is no predetermined budget, so the set of alternative policies is

$$P = \{(p_1, ..., p_n) \in \Re_+^n\}.$$

Assume that transfers are funded by a uniform tax on all districts $T = \sum_{i=1}^n p_i/n$. Moreover, because of distortions and tax collection costs, the per district cost of this tax is $C(T)$ where $C' > 1$, $C'' > 0$ and $lim_{T\to\infty} C'(T) = \infty$. Suppose that

$$V_i(p_1, ..., p_n) = p_i - C(T).$$

Apply the legislative bargaining model discussed in class to make a prediction of what will happen in this situation.

4. Consider the lobbying as legislative subsidy model discussed in class. Assume that the legislator's utility function is

$$U(P_a, P_o) = \alpha \ln P_a + (1 - \alpha) \ln P_o.$$

(i) Solve for the optimal choice of (P_a, P_o) without lobbying.

(ii) Solve for the optimal choice of (P_a, P_o) with legislative subsidy $s > 0$ to the legislator's effort for issue a. What happens to the legislator's effort for issue a as s increases?

(iii) How does the increase in P_a created by the subsidy depend upon the legislator's preference parameter α?

5. Consider a group of 9 individuals who have to choose one of three alternatives $\{a_1, a_2, a_3\}$. Citizens rankings of the alternatives take one of three possible forms. These three rankings are labelled I, II, and III, and are illustrated in the following table.

I	II	III
4	3	2
a_1	a_2	a_3
a_3	a_3	a_2
a_2	a_1	a_1

The numbers in the second row are the number of citizens who have each of the three rankings. Identify the alternative that the group would choose if they used the following voting rules: i) plurality rule; ii) majority rule with run offs; iii) single transferable vote; iv) Borda count; and v) approval voting. Assume throughout that individuals vote sincerely and that, under approval voting, they approve of their top two alternatives.

6. Consider a group of 8 individuals who have to choose one of three alternatives $\{a_1, a_2, a_3\}$. Citizens rankings of the alternatives take one of three possible forms. These three rankings are labelled I, II, and III, and are illustrated in the following table.

I	II	III
3	3	2
a_1	a_2	a_3
a_2	a_3	a_1
a_3	a_1	a_2

The numbers in the second row are the number of citizens who have each of the three rankings.

(i) Identify the alternative that the group would choose if they used the single transferable vote and all individuals truthfully reported their rankings.

(ii) Show that one of the citizens with ranking II could achieve a preferred outcome by misreporting his ranking.

PROBLEM SET #3

1. Consider a society of n citizens which has to decide whether or not to make a policy change. Suppose that the citizens can be divided into gainers and losers from the change. Let n_G denote the number of gainers and $n - n_G$ the number of losers. Suppose that gainers gain G from the change and that losers lose L. Assume that $0 < G < L$.

(i) Assuming a Utilitarian social welfare function, compute the critical value of n_G above which the policy change will increase social welfare.

(ii) Assuming that the citizens decide whether or not to make the change in a referendum, compute the critical value of n_G above which the policy change will be made. (You may assume that everyone votes and that the change is made only if it gets a majority of votes).

(iii) Assume that the decision of whether or not to make a policy change is delegated to an elected leader. Suppose that $n_G > n/2$ and that the leader is a gainer. Further suppose that the losers form an interest group that can offer campaign contributions to the leader. This interest group behaves so as to maximize the losers' collective payoff. If the interest group gives contribution C, this collective payoff is $-(n - n_G)L - C$ if the change is made and $-C$ if the change is not made. The leader's payoff if he is given contribution C is $G + \gamma C$ if the change is made and γC if the change is not made, where $\gamma \in (0, 1)$. Assuming that the interaction between the interest group and leader is as modeled in the common agency model, find the critical value of n_G above which the policy change will be made.

2. Consider the following model. There are N citizens and two periods. There are two goods - a consumption good x and labor l. Each period, individuals are endowed with 1 unit of labor. The population is equally divided into two ability types - low and high. Let $a_{L\tau}$ denote the wage of low-ability citizens in period τ and $a_{H\tau}$ that of high ability citizens. Citizens care only about their consumption and are risk neutral, so that in each period $u(x, l) = x$. Citizens do not discount across periods.

There is a government. In each period τ, the government sets the parameters of a linear income tax - a tax rate $t_\tau \in [0, 1]$ and a uniform transfer T_τ (which can be negative). In addition, in period 1, the government has to decide whether to undertake a discrete public investment which costs C. The government must balance its budget in each period. In period 1, $a_{L1} = a_L$ and $a_{H1} = a_H$, where $a_L < a_H$. If the public investment is not undertaken, individuals' second period wages just equal those in the first period. If the investment is undertaken, then it raises the wage of high-ability individuals from a_H to $a_H + \delta$ where $\delta > 0$.

(i) Under what conditions will not undertaking the public investment be Pareto inefficient? Explain carefully.

(ii) Using the citizen-candidate model, construct a political equilibrium in which the public investment is not undertaken even when not undertaking it is Pareto inefficient.

3. Consider the Debt example discussed in class.

(i) Compute the maximum discounted expected payoff that Republican politicians can obtain given the available policies.

(ii) Compute the maximum discounted expected payoff that Democrat politicians can obtain given the available policies.

(iii) Graph the utility possibility set associated with the available policies. (You should consider the full two period discounted utilities).

(iv) Compute the expected discounted payoffs for Democrat and Republican politicians associated with the political equilibrium described in class and illustrate these payoffs on your utility possibility set graph.

(v) Compute the expected discounted payoffs for Democrat and Republican politicians associated with the political equilibrium that would arise if the government were constitutionally required to run a balanced budget (i.e., no debt) and illustrate these payoffs on your utility possibility set graph. (You may continue to assume that Democrats hold the majority in period 1 and that each party holds the majority in period 2 with probability $\frac{1}{2}$).

4. Consider the simple districting model discussed in class with 3 districts.

(i) Prove that $V(j; i_m(V)) \geq 1/2$ is equivalent to

$$V \geq V^*(j) \equiv \pi_D + \pi_I[\frac{1/2 - \pi_D(j)}{\pi_I(j)}],$$

as asserted in the notes.

(ii) Describe the optimal partisan districting plan (for the Democrats) when $(\tau, \varepsilon) = (0.25, 0.15)$ and $(\pi_D, \pi_I, \pi_R) = (0.25, 0.4, 0.35)$. Also, compute the probability that the Democrats will hold the majority of seats.

(iii) Construct a socially optimal districting plan for the parameter values used in part (ii) (Assume that policy depends on the ideology of the median legislator).

5. Consider the Districting model discussed in class with a continuum of districts. Assume that policy depends on the ideology of the mean legislator and that $(\pi_D, \pi_I, \pi_R) = (0.35, 0.3, 0.35)$. Describe a districting that generates the optimal seat vote curve. (A districting in the continuum case is a description of the fractions of voters $(\pi_D(j), \pi_I(j), \pi_R(j))$ in each district $j \in [0, 1]$. You may find it helpful to consult the appendix of the Coate and Knight (2007) paper.)

6. Consider Gerber's citizens' initiatives model discussed in class. Change the model by assuming that $x_p < x_i < x_v$ and solve for the equilibrium outcome.

7. Consider the informative advertising model discussed in class. Suppose that both parties' candidates are qualified and that the Democrat candidate spends C_D on campaign advertising and the Republican spends C_R. Prior to voting, each voter must be in one of four informational states: i) he has seen ads from both candidates and thus knows they are both qualified; ii) he has seen an ad from the Democrat candidate but not from the Republican; iii) he has seen an ad from the Republican candidate but not from the Democrat; and iv) he has seen ads from neither candidate. The fraction of voters in state i) is $\lambda(C_D) \cdot \lambda(C_R)$; the fraction in state ii) is $\lambda(C_D) \cdot (1 - \lambda(C_R))$; etc, etc. Assume that voters assign probability ρ to a candidate being qualified.when they have not seen an ad from that candidate. Show that the probability that the Democrat wins the election is

$$\pi(C_D, C_R; \rho) = \frac{1}{2} + \frac{\frac{(1-\rho)\delta}{2\beta}}{2\varepsilon}(\lambda(C_D) - \lambda(C_R)).$$