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A Political Economy Theory of Fiscal Policy and Unemployment*

Abstract

This paper presents a political economy theory of fiscal policy and unemployment. The underlying economy is one in which unemployment can arise but can be mitigated by tax cuts and increases in public production. Such policies are fiscally costly, but can be financed by issuing government debt. Policy decisions are made by a legislature consisting of representatives from different political districts. With the available policies, it is possible for the government to completely eliminate unemployment in the long run. However, with political decision making, the economy always has unemployment. Unemployment is higher when the private sector experiences negative shocks. When these shocks occur, the government employs debt-financed fiscal stimulus plans which involve both tax cuts and public production increases. When the private sector is healthy, the government contracts debt until it reaches a floor level. Unemployment levels are weakly increasing in the economy's debt level, strictly so when the private sector experiences negative shocks. Conditional on the level of workers employed, the mix of public and private output is distorted.

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1 Introduction

An important role for fiscal policy is the mitigation of unemployment and stabilization of the economy.[1] Despite scepticism from some branches of the economics profession, politicians and policy-makers tend to be optimistic about the potential fiscal policy has in this regard. Around the world, countries facing downturns continue to pursue a variety of fiscal strategies, ranging from tax cuts to public works projects. Nonetheless, politicians' willingness to use fiscal policy to aggressively fight unemployment is tempered by high levels of debt. The main political barrier to deficit-financed tax cuts and public spending increases appears to be concern about the long-term burden of high debt.

This extensive practical experience with fiscal policy raises a number of basic positive public finance questions. In general, how do employment concerns impact the setting of taxes and public spending? When will government employ fiscal stimulus plans? What determines the size of these plans and how does this depend upon the economy's debt position? What will be the mix of tax cuts and public spending increases in stimulus plans? What will be the overall effectiveness of fiscal policy in terms of reducing unemployment?

This paper presents a political economy theory of the interaction between fiscal policy and unemployment that sheds light on these questions. The economic model underlying the theory is one in which unemployment can arise but can be mitigated by tax cuts and public spending increases. Such policies are fiscally costly, but can be financed by issuing debt. The political model assumes that policy decisions are made in each period by a legislature consisting of representatives from different political districts. Legislators can transfer revenues back to their districts which creates a political friction. The theory combines the economic and political models to provide a positive account of the simultaneous determination of fiscal policy and unemployment.

The political model underlying the theory follows the approach in our previous work (Battaglini and Coate 2007, 2008). The economic model is novel to this paper. It features a public and private sector.[2] The private sector consists of entrepreneurs who hire workers to produce a private good. The public sector hires workers to produce a public good. Public production is financed by a tax

[1] For an informative discussion of this role see Auerbach, Gale, and Harris (2010).

[2] In this sense, the model is similar to those used in that strand of the macroeconomics literature investigating the aggregate implications of changes in public sector employment, public production, etc. Examples include Ardagna (2007), Economides, Philippoulos, and Vassilatos (2013), Linnemann (2009), and Pappa (2009).

on the private sector. The government can also borrow and lend in the bond market. The private sector is affected by exogenous shocks (oil price hikes, for example) which impact entrepreneurs' demand for labor. Unemployment can arise because of a downwardly rigid wage. In the presence of unemployment, reducing taxes increases private sector hiring, while increasing public production creates public sector jobs. Thus, tax cuts and increases in public production reduce unemployment. However, both actions are costly for the government.

With the available policies, it is possible for the government to completely eliminate unemployment in the long run. However, with political decision making, the economy will always have unemployment under our assumptions. When the private sector experiences negative shocks, unemployment increases. When these shocks occur, government mitigates unemployment with stimulus plans that are financed by increases in debt. These equilibrium stimulus plans involve both tax cuts and increases in public production. When choosing such plans, the government balances the benefits of reducing unemployment with the costs of distorting the private-public output mix. This means that stimulus plans do not achieve the maximum possible reduction in unemployment and that the multiplier impacts of tax cuts and public production increases are not equalized. In normal times, when the private sector is not experiencing negative shocks, the government reduces debt until it reaches a floor level. At all times, the private-public output mix is distorted relative to the first best. Unemployment is weakly increasing in the government's debt level, strictly so when the private sector experiences negative shocks.

The theory has two unambiguous qualitative implications. The first is that the dynamic pattern of debt is counter-cyclical. This implication also emerges from other theories of fiscal policy, so there is nothing particularly distinctive about it. Some empirical support for this prediction already exists (see, for example, Barro 1986). The second implication is that, ceteris paribus, the larger an economy's pre existing debt level, the higher will be its unemployment rate. This implication should be distinguished from the positive correlation between contemporaneous debt and unemployment that arises from the fact that both are counter-cyclical. The underlying mechanism is that an economy's pre-existing debt level constrains its stimulus efforts. We are not aware of any other theoretical work that links pre-existing debt and unemployment in this way and so we believe this to be a novel prediction. While some prior evidence in favor of this prediction exists, the empirical relationship between debt and unemployment has attracted surprisingly little attention.[3] We thus augment existing evidence with a preliminary analysis of recent panel data from a group of OECD countries. We also use this data to provide preliminary support for another idea suggested by the theory: namely, that the volatility of employment levels should be positively correlated with debt.

While there is a vast theoretical literature on fiscal policy, we are not aware of any work that systematically addresses the positive public finance questions that motivate this paper. Neoclassical theories of fiscal policy, such as the tax smoothing approach, assume frictionless labor markets and thus abstract from unemployment. Traditional Keynesian models incorporate unemployment and allow consideration of the multiplier effects of changes in government spending and taxes. However, these models are static and do not incorporate debt and the costs of debt financing.[4] This limitation also applies to the literature in optimal taxation which has explored how optimal policies are chosen in the presence of involuntary unemployment.[5] The modern new Keynesian literature with its sophisticated dynamic general equilibrium models with sticky prices typically treats fiscal policy as exogenous.[6] Papers in this tradition that do focus on fiscal policy, analyze how government spending shocks impact the economy and quantify the possible magnitude of multiplier effects.[7]

The novelty of our questions and model not withstanding, the basic forces driving the dynamics of debt in our theory are similar to those arising in our previous work on the political determination of fiscal policy in the tax smoothing model (Battaglini and Coate 2008 and Barshegyan, Battaglini, and Coate 2013).[8] In the tax smoothing model the government must finance its spending with distortionary taxes but can use debt to smooth tax rates across periods. The need to smooth is created by shocks to government spending needs as a result of wars or disasters (as in Battaglini and Coate) or by cyclical variation in tax revenue yields due to the business cycle (as in Barshegyan,

[3] Exceptions are Bertola (2011), Fedeli and Forte (2011) and Fedeli, Forte, and Ricchi (2012). We discuss this evidence in Section 4.

[4] For a nice exposition of the traditional Keynesian approach to fiscal policy see Peacock and Shaw (1971). Blinder and Solow (1973) discuss some of the complications associated with debt finance and extend the IS-LM model to try to capture some of these.

[5] This literature includes papers by Bovenberg and van der Ploeg (1996), Dreze (1985), Marchand, Pestieau, and Wibaut (1989), and Roberts (1982).

[6] See, for example, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003).

[7] See, for example, Christiano, Eichenbaum, and Rebelo (2009), Hall (2009), Mertens and Ravn (2010), and Woodford (2010).

[8] For other political economy models of debt see Alesina and Tabellini (1990), Cukierman and Meltzer (1989), Persson and Svensson (1989), and Song, Storesletten, and Zilibotti (2012).

Battaglini, and Coate). With political determination, debt exhibits a counter-cyclical pattern, going up when the economy experiences negative shocks and back down when it experiences positive shocks. However, even after repeated positive shocks, debt never falls below a floor level. This reflects the fact that after a certain point legislators find it more desirable to transfer revenues back to their districts than to devote them to further debt decummulation. These basic lessons apply in our model of unemployment. This reflects the fact that debt plays a similar economic role, allowing the government to smooth the distortions arising from a downwardly rigid wage across periods.

Addressing the questions we are interested in requires a simple and tractable dynamic model. In creating such a model, we have made a number of strong assumptions. First, we employ a model without money and therefore abstract from monetary policy. This means that we cannot consider the important issue of whether the government would prefer to use monetary policy to achieve its policy objectives. Second, we obtain unemployment by simply assuming a downwardly rigid wage, as opposed to a more sophisticated micro-founded story.[9] This means that our analysis abstracts from any possible effects of fiscal policy on the underlying friction generating unemployment. Third, the source of cyclical fluctuations in our economy comes from the supply rather than the demand side. In our model, recessions arise because negative shocks to the private sector reduce the demand for labor. Labor market frictions prevent the wage from adjusting and the result is unemployment. This vision differs from the traditional and new Keynesian perspectives that emphasize the importance of shocks to consumer demand.[10] Finally, our model ignores any impact of fiscal policy on capital accumulation.

While these strong assumptions undoubtedly represent limitations of our analysis, we nonetheless feel that our model provides a useful framework in which to study the interaction between fiscal policy and unemployment. First, the model incorporates the two broad ways in which government can create jobs: *indirectly* by reducing taxes on the private sector, or *directly* through increasing public production. Second, the model allows consideration of two conceptually different types of activist fiscal policy: *balanced-budget policies* wherein tax cuts are financed by public

[9] There is a literature incorporating theories of unemployment into dynamic general equilibrium models (see Gali (1996) for a general discussion). Modelling options include matching and search frictions (Andolfatto 1996), union wage setting (Ardagna 2007), and efficiency wages (Burnside, Eichenbaum, and Fisher 1999).

[10] In the new Keynesian literature demand shocks are created by stochastic discount rates (see, for example, Christiano, Eichenbaum, and Rebelo (2009)).

spending decreases or visa versa, and *deficit-financed policies* wherein tax cuts and/or spending increases are financed by increases in public debt. Third, the mechanism by which taxes influence private sector employment in the model is consonant with arguments that are commonplace in the policy arena. For example, the main argument behind objections to eliminating the Bush tax cuts for those making $250,000 and above, was that it would lead small businesses to reduce their hiring during a time of high unemployment. Fourth, the mechanism by which high debt levels are costly for the economy also captures arguments that are commonly made by politicians and policy-makers. Higher debt levels imply larger service costs which require either greater taxes on the private sector and/or lower public spending. These policies, in turn, have negative consequences for jobs and the economy.

The organization of the remainder of the paper is as follows. Section 2 outlines the model. Section 3 describes equilibrium fiscal policy and unemployment. Section 4 develops and explores the empirical implications of the theory, and Section 5 concludes.

2 Model

The environment We consider an infinite horizon economy in which there are two final goods; a private good x and a public good g. There are two types of citizens; entrepreneurs and workers. Entrepreneurs produce the private good by combining labor l with their own effort. Workers are endowed with 1 unit of labor each period which they supply inelastically. The public good is produced by the government using labor. The economy is divided into m political districts, each a microcosm of the economy as a whole.

There are n_e entrepreneurs and n_w workers where $n_e + n_w = 1$. Each entrepreneur produces with the Leontief production technology $x = A_\theta \min\{l, \epsilon\}$ where ϵ represents the entrepreneur's effort and A_θ is a productivity parameter. The idea underlying this production technology is that when an entrepreneur hires more workers he must put in more effort to manage them. The productivity parameter varies over time, taking on one of two values A_L (low) and A_H (high) where A_L is less than A_H. The probability of high productivity is α. The public good production technology is $g = l$.

A worker who consumes x units of the private good obtains a per period payoff $x + \gamma \ln g$ when the public good level is g. Here, the parameter γ measures the relative value of the public good. Entrepreneurs' per period payoff function is $x + \gamma \ln g - \xi\epsilon^2/2$ where the third term represents the disutility of providing entrepreneurial effort. All individuals discount the future at rate β.

There are markets for the private good and labor. The private good is the numeraire. The wage is denoted ω and the labor market operates under the constraint that the wage cannot go below an exogenous minimum $\underline{\omega}$.[11] This friction is the source of unemployment. The minimum wage $\underline{\omega}$ is assumed to be less than A_L. There is also a market for risk-free one period bonds. The assumption that citizens have quasi-linear utility implies that the equilibrium interest rate on these bonds is $\rho = 1/\beta - 1$.

To finance its activities, the government taxes entrepreneurs' incomes at rate τ. It can also borrow and lend in the bond market. Government debt is denoted by b and new borrowing by b'. The government is also able to distribute surplus revenues to citizens via district-specific lump sum transfers. Let s_i denote the transfer going to the residents of district $i \in \{1, ..., m\}$.

Market equilibrium At the beginning of each period, the productivity state is revealed. The government repays existing debt and chooses the tax rate, public good, new borrowing, and transfers. It does this taking into account how its policies impact the market and the need to balance its budget.

To understand how policies impact the market, assume the state is θ, the tax rate is τ, and the public good level is g. Given a wage rate ω, each entrepreneur chooses hiring, the input, and effort, to maximize his utility

$$\max_{(l,\epsilon)} (1-\tau)(A_\theta \min\{l, \epsilon\} - \omega l) - \xi \frac{\epsilon^2}{2}. \tag{1}$$

Obviously, the solution involves $\epsilon = l$. Substituting this into the objective function and maximizing with respect to l reveals that $l = (1-\tau)(A_\theta - \omega)/\xi$. Aggregate labor demand from the private sector is therefore $n_e(1-\tau)(A_\theta - \omega)/\xi$. Labor demand from the public sector is g and labor supply

[11] We make this assumption to get a simple and tractable model of unemployment. While $\underline{\omega}$ could be literally interpreted as a statutory minimum wage, what we are really trying to capture are the sort of rigidities identified in the survey work of Bewley (1999). The assumption of some type of wage rigidity is common in the macroeconomics literature (see, for example, Blanchard and Gali (2007), Hall (2005), and Michaillat (2012)) and a large empirical literature investigates the extent of wage rigidity in practice (see, for example, Barwell and Schweitzer (2007), Dickens et al (2007), and Holden and Wulfsberg (2009)).

is n_w.[12] Setting demand equal to supply, the market clearing wage is

$$\omega = A_\theta - \xi(\frac{n_w - g}{n_e(1-\tau)}). \tag{2}$$

The minimum wage will bind if this wage is less than $\underline{\omega}$. In this case, the equilibrium wage is $\underline{\omega}$ and the unemployment rate is

$$u = \frac{n_w - g - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi}{n_w}. \tag{3}$$

To sum up, in state θ with government policies τ and g, the equilibrium wage rate is

$$\omega_\theta = \begin{cases} \underline{\omega} & \text{if } A_\theta \leq \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}) \\ A_\theta - \xi(\frac{n_w - g}{n_e(1-\tau)}) & \text{if } A_\theta > \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}) \end{cases} \tag{4}$$

and the unemployment rate is

$$u_\theta = \begin{cases} \frac{n_w - g - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi}{n_w} & \text{if } A_\theta \leq \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}) \\ 0 & \text{if } A_\theta > \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}). \end{cases} \tag{5}$$

When the minimum wage is binding, the unemployment rate is increasing in τ. Higher taxes cause entrepreneurs to put in less effort and this reduces private sector demand for workers. The unemployment rate is also decreasing in g because to produce more public goods, the government must hire more workers. When the minimum wage is not binding, the equilibrium wage is decreasing in τ and increasing in g.

Each entrepreneur earns profits of $\pi_\theta = (1-\tau)(A_\theta - \omega_\theta)^2/\xi$. Assuming he receives no government transfers and consumes his profits, an entrepreneur obtains a period payoff of

$$v_{e\theta} = \frac{(A_\theta - \omega_\theta)^2(1-\tau)^2}{2\xi} + \gamma \ln g. \tag{6}$$

Jobs are randomly allocated among workers and so each worker obtains an expected period payoff

$$v_{w\theta} = (1 - u_\theta)\omega_\theta + \gamma \ln g. \tag{7}$$

Again, this assumes that the worker receives no transfers and simply consumes his earnings.

[12] The model assumes that the government pays the same wage as do entrepreneurs and therefore makes no distinction between public and private sector wages. It therefore abstracts from the reality that public and private sector wages are not determined in the same way. For macroeconomic analysis focusing on this distinction see, for example, Fernandez-de-Cordoba, Perez, and Torres (2012).

Aggregate output of the private good is $x_\theta = n_e A_\theta(1-\tau)(A_\theta - \omega_\theta)/\xi$. Substituting in the expression for the equilibrium wage, we see that

$$x_\theta = \begin{cases} n_e A_\theta(1-\tau)(A_\theta - \underline{\omega})/\xi & \text{if } A_\theta \leq \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}) \\ A_\theta\,(n_w - g) & \text{if } A_\theta > \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}). \end{cases} \tag{8}$$

Observe that the tax rate has no impact on private sector output when the minimum wage constraint is not binding. This is because labor is inelastically supplied and as a consequence the wage adjusts to ensure full employment. A higher tax rate just leads to an offsetting reduction in the wage rate. However, when there is unemployment, tax hikes reduce private sector output because they lead entrepreneurs to reduce effort. Public good production has no effect on private output when there is unemployment, but reduces it when there is full employment.

The government budget constraint Having understood how markets respond to government policies, we can now formalize the government's budget constraint. Tax revenue is

$$R_\theta(\tau, \omega_\theta) = \tau(n_e \pi_\theta) = \tau n_e(1-\tau)(A_\theta - \omega_\theta)^2/\xi. \tag{9}$$

Total government revenue includes tax revenue and new borrowing and therefore equals $R_\theta(\tau, \omega_\theta) + b'$. The cost of public good provision and debt repayment is $\omega_\theta g + b(1+\rho)$. The budget surplus available for transfers is therefore

$$B_\theta(\tau, g, b', b, \omega_\theta) = R_\theta(\tau, \omega_\theta) + b' - (\omega_\theta g + b(1+\rho))\,. \tag{10}$$

The government budget constraint is that this budget surplus be sufficient to fund any transfers made, which requires that

$$B_\theta(\tau, g, b', b, \omega_\theta) \geq \sum_{i=1}^{m} s_i. \tag{11}$$

There is also an upper limit $\overline{b}$ on the amount of debt the government can issue. This limit is motivated by the unwillingness of borrowers to hold bonds that they know will not be repaid (i.e., it is a "natural debt limit" in the terminology of Aiyagari et al 2002). If, in steady state, the government were borrowing an amount b such that the interest payments exceeded the maximum possible tax revenues in the low productivity state; i.e., ρb exceeded $\max_\tau R_L(\tau, \underline{\omega})$, then, if productivity were low, it would be unable to repay the debt *even if it provided no public goods or transfers.* Borrowers would therefore be unwilling to lend more than $\max_\tau R_L(\tau, \underline{\omega})/\rho$. For technical reasons, it is convenient to assume that the upper limit $\overline{b}$ is equal to $\max_{\tau} R_L(\tau, \underline{\omega})/\rho - \varepsilon$, where $\varepsilon > 0$ can be arbitrarily small.

Political decision-making Government policy decisions are made by a legislature consisting of m representatives, one from each district. Each representative wishes to maximize the aggregate utility of the citizens in his district. In addition to choosing taxes, public goods, and borrowing, the legislature must also divide any budget surplus between the districts. The affirmative votes of m/q representatives are required to enact any legislation, where $q > 1$. Lower values of q mean that more legislators are required to approve legislation and thus represent more inclusive decision-making.

The legislature meets at the beginning of the period after the productivity state θ is known. The decision-making process follows a simple sequential protocol. At stage $j = 1, 2, ...$ of this process, a representative is randomly selected to make a proposal to the floor. A proposal consists of policies (τ, g, b') and district-specific transfers $(s_i)_{i=1}^m$ satisfying the constraints that transfer spending $\sum_i s_i$ does not exceed the budget surplus $B_\theta(\tau, g, b', b, \omega_\theta)$ and new borrowing b' does not exceed the debt limit $\overline{b}$. If the proposal receives the votes of m/q representatives, then it is implemented and the legislature adjourns until the following period. If the proposal does not pass, then the process moves to stage $j+1$, and a representative is selected again to make a new proposal.[13]

3 Equilibrium fiscal policy and unemployment

Following the analysis in Battaglini and Coate (2008), it can be shown that in productivity state θ with initial debt level b, the equilibrium levels of taxation, public good spending, and new borrowing $\{\tau_\theta(b), g_\theta(b), b'_\theta(b)\}$ solve the maximization problem:

$$\max_{(\tau,g,b')} \left\{ \begin{array}{c} qB_\theta(\tau, g, b', b, \omega_\theta) + n_e v_{e\theta} + n_w v_{w\theta} + \beta E V_{\theta'}(b') \\ \\ s.t.\ B_\theta(\tau, g, b', b, \omega_\theta) \geq 0 \ \&\ b' \leq \overline{b} \end{array} \right\}, \tag{12}$$

where $V_{\theta'}(b')$ is equilibrium aggregate lifetime citizen expected utility in state θ' with debt level b'. The equilibrium level of spending on transfers is equal to the budget surplus associated with the

[13] This process may either continue indefinitely until a proposal is chosen, or may last for a finite number of stages as in Battaglini and Coate (2008): the analysis is basically the same. In Battaglini and Coate (2008) it is assumed that in the last stage, one representative is randomly picked to choose a policy; this representative is then required to choose a policy that divides the budget surplus evenly between districts.

policies $\{\tau_\theta(b), g_\theta(b), b'_\theta(b)\}$, which is $B_\theta(\tau_\theta(b), g_\theta(b), b'_\theta(b), b, \omega_\theta)$. The equilibrium value functions $V_H(b)$ and $V_L(b)$ in problem (12) are defined recursively by the equations:

$$V_\theta(b) = B_\theta(\tau_\theta(b), g_\theta(b), b'_\theta(b), b, \omega_\theta) + n_e v_{e\theta} + n_w v_{w\theta} + \beta E V_{\theta'}(b'_\theta(b)) \tag{13}$$

for $\theta \in \{L, H\}$. Representatives' value functions, which reflect only aggregate utility in their respective districts, are equal to $V_H(b)/m$ and $V_L(b)/m$.[14]

A convenient short-hand way of understanding the equilibrium is to imagine that in each period a minimum winning coalition (mwc) of m/q representatives is randomly chosen and that this coalition collectively chooses policies to maximize *its* aggregate utility (as opposed to society's). Problem (12) reflects the coalition's maximization problem. Recall that $v_{e\theta}$ and $v_{w\theta}$ denote, respectively, entrepreneur and worker per period payoffs net of transfers. Thus, if q were equal to 1 so that legislation required unanimous approval, the objective function in (12) would exactly equal aggregate societal utility. In this case (12) would correspond to the planner's problem for this economy. Since q exceeds 1, problem (12) differs from a planning problem in that extra weight is put on the surplus available for transfers. This extra weight reflects the fact that transfers are shared only among coalition members. Because membership in the mwc is random, all representatives are ex ante identical and have a common value function given by (13) (divided by $1/m$). In what follows, we will use this way of understanding the equilibrium and speak *as if* a randomly drawn mwc is choosing policy in each period.

The equilibrium policies are characterized by solving problem (12). It will prove instructive to break down the analysis of this problem into two parts. First, we study the associated static problem. Thus, we fix new borrowing b' and assume that the mwc faces an exogenous revenue requirement equal to $b' - (1+\rho)b$. Then, we endogenize the revenue requirement by studying the choice of debt.

3.1 The static problem

The static problem for the mwc is to choose a tax rate τ and a level of public good g to maximize its collective utility given that revenues net of public production costs must cover a revenue

[14] A political equilibrium amounts to a set of policy functions that solve (12) given the equilibrium value functions, and value functions that satisfy (13) given the equilibrium policies. A political equilibrium is *well-behaved* if the associated value functions $V_L(b)$ and $V_H(b)$ are concave in b. Following the approach in Battaglini and Coate (2008), it can be shown that a well-behaved political equilibrium exists. The analysis will focus on well-behaved equilibria and we will refer to them simply as equilibria. More explanation of this characterization of equilibrium and a discussion of the existence of an equilibrium can be found in our working paper, Battaglini and Coate (2011).

requirement r and that net revenues in excess of r finance transfers to the districts of coalition members. Using the definition of the budget surplus function in (10) and the assumption that $r = b' - (1+\rho)b$, the mwc's static problem can be posed as:

$$\max_{(\tau,g)} \left\{ \begin{array}{c} q\left(R_\theta(\tau,\omega_\theta) - \omega_\theta g - r\right) + n_e v_{e\theta} + n_w v_{w\theta} \\ s.t.\ R_\theta(\tau,\omega_\theta) - \omega_\theta g \geq r \end{array} \right\}. \tag{14}$$

Since the difference between new borrowing and debt repayment (i.e., $b' - (1+\rho)b$) could in principle be positive or negative, the revenue requirement r can be positive or negative.

The first point to note about the problem is that the mwc will always set taxes sufficiently high so that the equilibrium wage equals $\underline{\omega}$. As noted earlier, taxes are non-distortionary when the wage exceeds $\underline{\omega}$ and the mwc has the ability to target transfers to its members. Thus, if the wage exceeded $\underline{\omega}$, there would be an increase in the mwc's collective utility if it raised taxes and used the additional tax revenues to fund transfers. Combining this observation with equations (6), (7), and (8), allows us to write problem (14) as:

$$\max_{(\tau,g)} \left\{ \begin{array}{c} x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + (q-1)\left(R_\theta(\tau,\underline{\omega}) - \underline{\omega} g\right) - qr \\ s.t.\ R_\theta(\tau,\underline{\omega}) - \underline{\omega} g \geq r \ \&\ g + \frac{x_\theta(\tau)}{A_\theta} \leq n_w \end{array} \right\}, \tag{15}$$

where $x_\theta(\tau)$ is the output of the private good when the tax rate is τ and the wage rate is $\underline{\omega}$ (see the top line of (8)).

Problem (15) has a simple interpretation. The objective function is the mwc's collective surplus.[15] The first inequality is the *budget constraint*: it requires that the mwc have sufficient net revenues to meet the revenue requirement under the assumption that the wage is $\underline{\omega}$. The second inequality is the *resource constraint*: it requires that the demand for labor at wage $\underline{\omega}$ is less than or equal to the number of workers n_w. This constraint ensures that the equilibrium wage is indeed $\underline{\omega}$.

A diagrammatic approach will be helpful in explaining the solution to problem (15). Without loss of generality, we assume here that r is less than or equal to the maximum possible tax revenue which is $R_\theta(1/2, \underline{\omega})$.[16] We also assume that unemployment would result if the government faced

[15] The expression for the surplus generated by $x_\theta(\tau)$ (the first two terms) reflects the fact that the surplus associated with the private good consists of the consumption benefits it generates less the costs associated with the entrepreneurial effort necessary to produce it.

[16] The revenue maximizing tax rate is 1/2 and the maximum revenue requirement is $n_e A_\theta(A_\theta - \underline{w})/4\xi$. Of

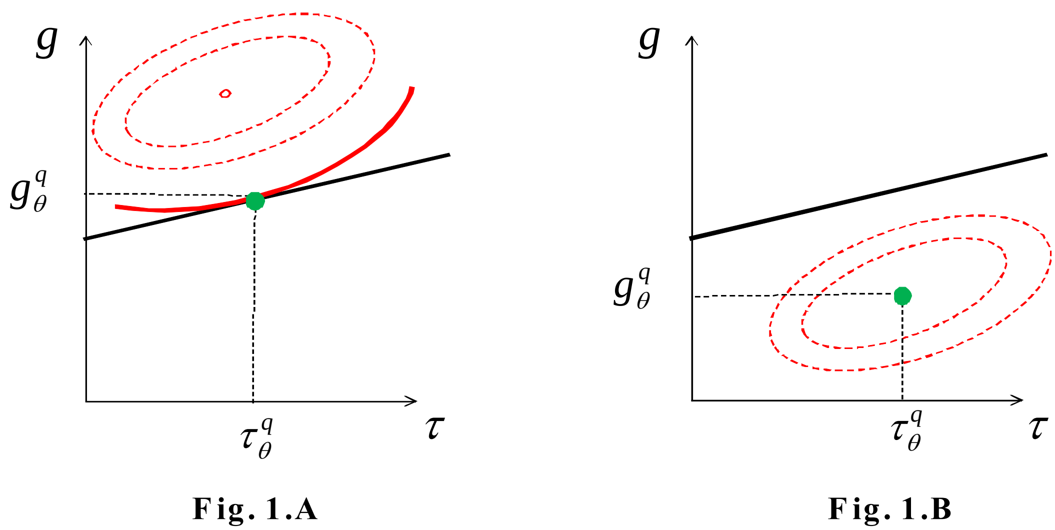

Figure 1:

the maximal revenue requirement.[17] To understand our diagrammatic approach, consider first Fig. 1.A and 1.B, where we ignore the budget constraint. The tax rate is measured on the horizontal axis and the public good on the vertical. In both figures, the upward sloping line is the frontier of the resource constraint. Using the expression for $x_\theta(\tau)$ from (8), this line is described by

$$g = n_w - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi. \tag{16}$$

At points along this line, there is full employment at the wage $\underline{\omega}$ and we therefore refer to it as the *full-employment line.* The resource constraint implies that policies must be on or below this line and points below are associated with unemployment. The other curves in the figures represents the mwc's *indifference curves.* Each curve satisfies for some target utility level U, the equation

$$x_\theta(\tau) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma\ln g + (q-1)\left(R_\theta(\tau,\underline{\omega}) - \underline{\omega}g\right) = U. \tag{17}$$

As illustrated, the mwc's preferences exhibit an interior satiation point in (τ, g) space. Two cases are possible. The first, represented in Figure 1.A, is where the satiation point is outside the

course, if r were higher than this level, the problem would have no solution. In the dynamic model, however, this case will never arise.

[17] If the government faces the maximal revenue requirement it will set the tax rate equal to 1/2 and provide no public good. Private sector employment will be $n_e(A_\theta - \underline{\omega})/2\xi$ and there will be no public sector employment. Thus, this assumption amounts to the requirement that n_w exceeds $n_e(A_\theta - \underline{\omega})/2\xi$.

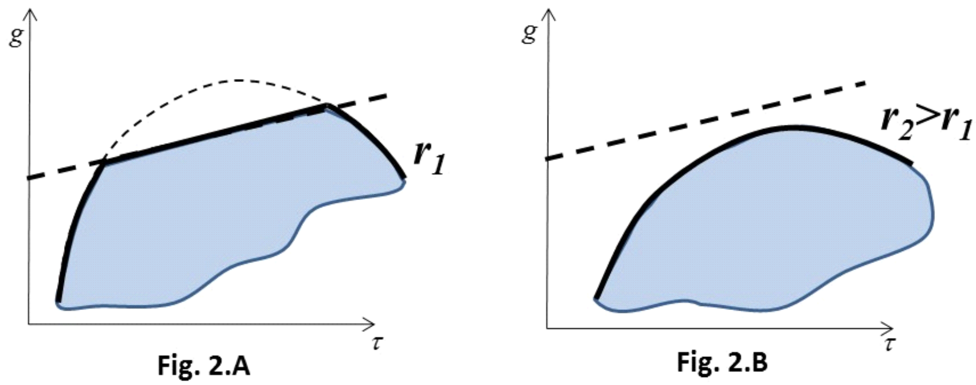

Figure 2:

resource constraint. In this case the optimal policies for the mwc ignoring the budget constraint (hereafter referred to as the *unconstrained optimal policies* and denoted $(\tau_\theta^q, g_\theta^q)$) lie at the point of tangency between the indifference curve and the full employment line. The second case, represented in Figure 1.B, is where the satiation point is inside the full employment line. In this case, the unconstrained optimal policies $(\tau_\theta^q, g_\theta^q)$ are just the satiation point. The mwc's preferred tax rate is sufficiently high and its preferred public good level sufficiently low, that unemployment arises. Intuitively, this case arises when the mwc's desire for surplus revenues is sufficiently strong that it overwhelms the costs of reduced aggregate consumption of the private and public good. This requires that q is significantly larger than 1.

To complete the description of problem (15), we need to add to this diagrammatic representation the government's budget constraint. The frontier of the budget constraint associated with revenue requirement r is given by

$$g = \frac{R_\theta(\tau, \underline{\omega})}{\underline{\omega}} - \frac{r}{\underline{\omega}}. \tag{18}$$

We refer to this as the *budget line*. The budget constraint requires that policies must be on or below this line and points below are associated with positive transfers. Each budget line is hump shaped, with peak at $\tau = 1/2$. Increasing the revenue requirement shifts down the budget line but does not change the slope. Figure 2 illustrates the budget line associated with two different revenue requirements. The feasible set of (τ, g) pairs for the mwc's problem are those that lie below both the budget and full employment lines. This set is represented by the gray areas in

Figure 2. Observe that the feasible set is (weakly) convex which makes the problem well-behaved. As the revenue requirement is raised, the set of policies for which full employment results shrinks. For sufficiently high revenue requirements it is not possible to achieve full employment (as in Fig. 2.B).

Before using this diagrammatic apparatus to explain the mwc's optimal policies, we make two important assumptions on our parameter values. Our first assumption implies that in both productivity states we are in the case illustrated in Fig. 1.B; that is, the mwc's preferred tax rate τ_θ^q is sufficiently high and its preferred public good level g_θ^q sufficiently low that unemployment arises.

Assumption 1

$$n_e \left[\frac{qA_H - (q-1)\underline{\omega}}{\xi(2q-1)} \right] + \frac{\gamma}{(q-1)\,\underline{\omega}} < n_w.$$

This condition is obtained by solving for what τ_H^q and g_H^q must be if the resource constraint is not binding and then imposing that at these values total employment is less than n_w. The condition holding in the high productivity state implies that it holds in the low productivity state. The purpose of this assumption is to simply to streamline the presentation. Dealing with both the cases illustrated in Figure 1 makes the analysis very taxonomic and thus much more challenging to follow. Readers interested in seeing what happens when Assumption 1 is not satisfied are referred to our working paper Battaglini and Coate (2014).

Our second assumption implies that in both productivity states tax revenues at rate τ_θ^q exceed the cost of providing public good level g_θ^q.

Assumption 2

$$n_e \left[\frac{(A_L q - \underline{\omega}(q-1))(A_L(q-1) - \underline{\omega} q)}{\xi(2q-1)^2} \right] > \frac{\gamma}{(q-1)}$$

The condition is obtained by solving for τ_L^q and g_L^q and then imposing that $R_L(\tau_L^q, \underline{\omega})$ exceeds $\underline{\omega} g_L^q$. It is straightforward to show that $R_H(\tau_H^q, \underline{\omega}) - \underline{\omega} g_H^q$ exceeds $R_L(\tau_L^q, \underline{\omega}) - \underline{\omega} g_L^q$, so that the condition holding in the low productivity state implies that it holds in the high productivity state. The role of this assumption, which will become clear later in the paper, is to guarantee that the equilibrium level of debt is positive. Note that, ceteris paribus, both Assumption 1 and 2 are more likely to hold for higher values of q. However, the reader should rest assured that they do not require unreasonably high values of q. For the case of majority rule (i.e., $q = 2$), it is easy to find sensible parameter values for which both Assumptions hold.

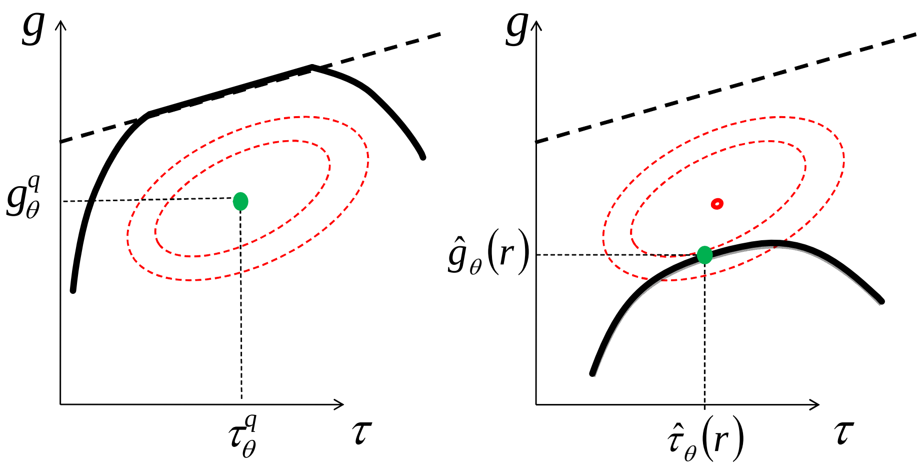

Fig. 3.A **Fig. 3.B**

Figure 3:

We are now ready to explain the mwc's optimal policies. Define r_θ^q to be the revenue requirement equal to $R_\theta(\tau_\theta^q, \underline{\omega}) - \underline{\omega} g_\theta^q$. This will be positive under Assumption 2. When r is below r_θ^q the budget constraint is not binding and the mwc will choose the unconstrained optimal policies $(\tau_\theta^q, g_\theta^q)$ and use the surplus revenues $r_\theta^q - r$ to finance transfers to their districts. This case is illustrated in Fig. 3.A. While there will be unemployment in this range of revenue requirements, it will be independent of the exact value of r. Any increase in the revenue requirement will simply be accommodated by a reduction in transfers.

When r is higher than r_θ^q, the mwc's unconstrained optimal tax rate and public good level generate insufficient revenue to meet the revenue requirement. The budget constraint binds and the mwc must generate further revenue by reducing public good provision and raising taxes. The optimal policies lie at the tangency of the budget line and the indifference curve. These policies are denoted by $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ and are illustrated in Fig. 3.B. It can be shown that as the revenue requirement climbs above r_θ^q, the tax rate increases and the public production level decreases, so $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ moves to the South-East. Unemployment also increases.

Summarizing this discussion, we have the following description of the solution to the static problem.

Proposition 1 *Suppose that Assumptions 1 and 2 hold. Then, in productivity state θ if the revenue requirement r is less than r_θ^q, the optimal policies for the static problem are $(\tau_\theta^q, g_\theta^q)$ and the level of transfers is $r_\theta^q - r$. There will be unemployment but it will be independent of r. If r exceeds r_θ^q, the optimal policies are $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ and no transfers are made. In this range, an increase in the revenue requirement results in an increase in the tax rate, a decrease in the public good level, and an increase in unemployment.*

3.2 The choice of debt

We now bring debt back into the picture. Recalling that $b'_\theta(b)$ denotes equilibrium new borrowing, the revenue requirement implied by the equilibrium policies in state θ with initial debt level b will be $r_\theta(b) = (1+\rho)b - b'_\theta(b)$. The equilibrium tax rate, public good level, and level of transfers, will be the solutions to the static problem described in Proposition 1 associated with this revenue requirement. The task is thus to identify the revenue requirements that arise in equilibrium and this requires understanding the behavior of debt.

Intuitively, debt can be used in two ways by the mwc. First, if the existing debt level is low, the mwc can ramp up debt to finance transfers to coalition members. Such borrowing raises the revenue requirements for future mwcs which will reduce their expenditure on transfers. Given that members of the current mwc may not belong to future mwcs, increasing current transfers at the expense of those of future mwcs is always attractive. Second, the mwc can use debt to smooth distortions. By borrowing in low productivity states and paying down debt in high productivity states, the mwc can transfer revenues from times with robust private sector profits and high labor demand to times when the private sector is depressed. In bad times, the revenues transferred will reduce fiscal pressure and permit policy changes which reduce unemployment and raise public and private sector outputs. The benefits from these changes will exceed the costs associated with raising revenue to pay down debt in good times because the distortions created by tax increases and public good reductions are lower in good times. Comparing these two uses of debt, only the second will persist in the long run. The ramping up of debt to shift forward transfers can occur only once. After it has occurred, the economy's debt level will be sufficiently high to deter future mwcs from debt issues of a similar scale and purpose.

Our interest is in understanding the steady state behavior of debt. Given the presence of productivity shocks, this steady state will be stochastic. To be more precise, given the equilibrium policy functions, for any initial debt level b, let $H(b, b')$ be the probability that next period's debt level will be less than b'. Given a distribution $\psi_{t-1}(b)$ of debt at time $t-1$, the distribution at time t, $\psi_t(b')$, is equal to $\int_b H(b, b') d\psi_{t-1}(b)$. A distribution $\psi^*(b')$ is said to be an invariant distribution if $\psi^*(b')$ is equal to $\int_b H(b, b') d\psi^*(b)$. If it exists, the invariant distribution describes the steady state of the government's debt distribution. We now have:

Proposition 2 *Suppose that Assumptions 1 and 2 hold. There exists a floor debt level $b^q \in (r_L^q/\rho, \overline{b})$ such that the equilibrium debt distribution converges to a unique, non-degenerate, invariant distribution with full support on $[b^q, \overline{b}]$. The dynamic pattern of debt is counter-cyclical: the government expands debt when private sector productivity is low and contracts debt when productivity is high until it reaches the floor level b^q.*

The floor debt level b^q reflects the mwc's incentive to use debt to shift forward transfers. If the economy starts out with a debt level below b^q, the mwc will ramp it up to b^q in the first period and use the proceeds to fund transfers to coalition members.[18] By contrast, the counter-cyclical behavior of debt in steady state reflects the use of debt to smooth distortions. This smoothing, however, is limited by the unwillingness of the mwc to reduce debt below the floor level b^q. Intuitively, if the debt level were ever to go below b^q, this would activate the incentive for future mwcs to use debt to shift forward transfers. Thus, the current mwc divert surplus revenues to transfers rather than to paying down debt below b^q. As noted in the introduction, this general pattern is analogous to the results of Battaglini and Coate (2008) and Barshegyan, Battaglini, and Coate (2013) for the tax smoothing model. The debt level b^q depends on the fundamentals of the economy and can be characterized following the approach in Battaglini and Coate (2008), but these details are not central to our mission here.[19]

With this appreciation of the steady state behavior of debt, we can now understand the revenue requirements that will arise in equilibrium. Combining this information with Proposition 1 will then reveal the steady state behavior of taxes, public goods, transfers, and unemployment. Note first that higher debt levels can be shown to translate into higher revenue requirements for the government (i.e., $r_\theta(b)$ is increasing in b for each state θ). Thus, Proposition 2 implies that the range of revenue requirements arising in steady state in state θ are $[r_\theta(b^q), r_\theta(\overline{b})]$. It can also be

[18] Note that b^q must be positive since it exceeds r_L^q/ρ and Assumption 2 implies that r_L^q is positive.

[19] The formal characterization of the debt level b^q is provided in the proof of Proposition 2.

shown that $r_L(b^q)$ exceeds r_L^q, implying that, in the low productivity state, steady state revenue requirements always exceed r_L^q. By Proposition 1, this means that, in steady state, there will be no transfers in the low productivity state. Moreover, the tax rate and unemployment will be increasing in the debt level and public good provision will be decreasing. By contrast, $r_H(b^q)$ is less than r_H^q. This means that, in steady state, there will be transfers in the high productivity state when debt levels are in the lower range of the support. Moreover, it will only be for debt levels above the critical level satisfying $r_H(b) = r_H^q$, that the tax rate and unemployment will be increasing in the debt level and public good provision will be decreasing. Pulling together all these observations, we can establish:

Proposition 3 *Suppose that Assumptions 1 and 2 hold, then the following is true in steady state. There is always unemployment and, for any given debt level, unemployment is higher when private sector productivity is low than when it is high. Unemployment is weakly increasing in the economy's debt level, strictly so in the low productivity state and in the high productivity state for debt levels above a critical level. Similarly, tax rates are weakly increasing and public good levels are weakly decreasing in the economy's debt level, strictly so in the low productivity state and in the high productivity state for debt levels above a critical level.*

3.3 Equilibrium stimulus plans

Proposition 2 tells us that in the steady state of the political equilibrium, when private sector productivity is low, the government expands debt and the funds are used to mitigate unemployment. The government therefore employs fiscal stimulus plans, as conventionally defined. By studying the size of these stimulus plans and the changes in policy they finance, we obtain a positive theory of fiscal stimulus. More specifically, in the low productivity state, we can interpret $\rho b - r_L(b)$ as the magnitude of the stimulus, since this measures the amount of additional resources obtained by the government to finance fiscal policy changes (i.e., the debt increase $b'_L(b) - b$). An understanding of how the stimulus funds are used can be obtained by comparing the equilibrium tax and public good policies with the policies that would be optimal if the debt level were held constant.

The use of stimulus funds Fig. 4.A illustrates what happens. From Proposition 1, the policies that would be chosen if the debt level were held constant are $(\widehat{\tau}_L(\rho b), \widehat{g}_L(\rho b))$. The reduction in the revenue requirement made possible by the stimulus funds, shifts the budget line up and permits a new policy choice $(\widehat{\tau}_L(r_L(b)), \widehat{g}_L(r_L(b)))$. As discussed in Section 3.1, the tax rate is increasing

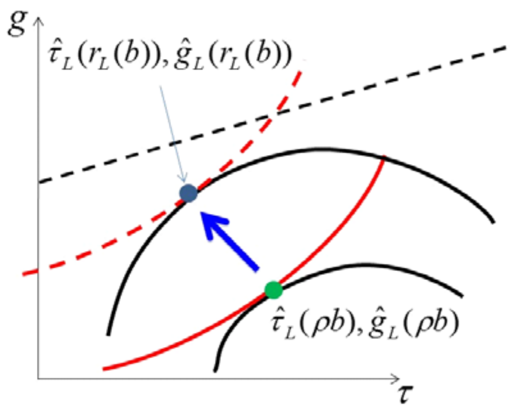

Fig. 4.A

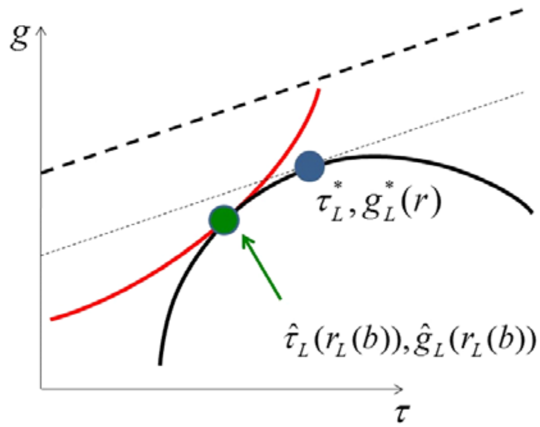

Fig. 4.B

Figure 4:

in the revenue requirement and public production is decreasing. Thus, we know that $\widehat{\tau}_L(r_L(b))$ is less than $\widehat{\tau}_L(\rho b)$ and that $\widehat{g}_L(r_L(b))$ exceeds $\widehat{g}_L(\rho b)$, implying that stimulus funds will be used for *both* tax cuts and increases in public production.[20]

Effectiveness and multipliers In terms of the effectiveness of equilibrium stimulus plans, the equilibrium policies will not typically minimize unemployment. The unemployment minimizing policies when the revenue requirement is r involve the tax rate τ_L^* at which the slope of the budget line is equal to the slope of the full employment line with associated public good level $g_L^*(r)$ given by (18) (see Fig. 4.B).[21] If $\widehat{\tau}_L(r_L(b))$ is less than τ_L^* (as in Fig. 4.B), then reducing the tax cut slightly and using the revenues to finance a slightly larger public production increase will produce a bigger reduction in unemployment. Conversely, if $\widehat{\tau}_L(r_L(b))$ exceeds τ_L^* then reducing the public production increase and using the revenues to finance a slightly larger tax cut will produce a bigger

[20] It should be stressed that the purpose of the tax cuts is to incentivize the private sector to hire more workers. This is logically distinct from the idea that tax cuts return purchasing power to citizens and stimulate demand, thereby creating jobs. Both types of arguments for tax cuts arise in the policy debate and it is important to keep them distinct. Similarly, the purpose of the increase in government spending is to hire more public sector workers, not to increase transfers to citizens. Notice that while the model allows the government to use stimulus funds to increase transfers, it chooses not to do so. Such transfers would have no aggregate stimulative effect because they must be paid for by future taxation. Taylor (2011) argues that the 2009 American Recovery and Reinvestment Act largely consisted of increases in transfers. Moreover, he argues that these transfer increases had little impact on household consumption since they were saved.

[21] In the Appendix, we show that $\tau_\theta^* = (A_\theta - 2\underline{\omega})/2(A_\theta - \underline{\omega})$. This discussion assumes that $g_\theta^*(r) = \frac{R_\theta(\tau_\theta^*, \underline{w})}{\underline{w}} - \frac{r}{\underline{w}}$ is non-negative. If this is not the case, the unemployment minimizing tax rate is such that $R_\theta(\tau, \underline{w}) = r$ and the associated public good level is 0.

reduction in unemployment. Both situations are possible depending on the parameters and the economy's debt level.[22] In the former case, legislators hold back from increasing taxes because, even though more jobs are created, the lost private output is more valuable than the additional public output. In the latter case, legislators hold back from reducing public production for the opposite reason.

One way of thinking about these results is in terms of multipliers. It is commonplace in the empirical literature to try to evaluate the multipliers associated with different stimulus measures.[23] The multiplier associated with a particular stimulus measure is defined to be the change in GDP divided by the budgetary cost of the measure. In our model, measuring GDP is more problematic than in the typical macroeconomic model because output is produced by both the private and public sectors, and there is no obvious way to value public sector output. Perhaps the simplest approach is to define GDP as equalling private sector output plus the cost of public production. With this definition, when there is unemployment, the public production multiplier is 1 and the tax cut multiplier is approximately $A_L / [(1 - 2\widehat{\tau}_L(r_L(b)))(A_L - \underline{\omega})]$. The tax cut multiplier will exceed the public production multiplier if $\widehat{\tau}_L(r_L(b))$ exceeds τ_L^* and be less than the public production multiplier if $\widehat{\tau}_L(r_L(b))$ is less than τ_L^*. The analysis illustrates why we should not expect the government to choose policies in such a way as to equate multipliers across instruments. Tax cuts and public production increases have different implications for the mix of public and private outputs. A further important point to note is that the tax multiplier is highly non-linear.[24] Tax cuts will be more effective the larger is the tax rate and the tax rate will be higher the larger the economy's debt level.

The magnitude of stimulus It is interesting to understand how the magnitude of the stimulus as measured by $\rho b - r_L(b)$ depends on the initial debt level b. Note first that as b approaches its maximum level $\overline{b}$, the size of the stimulus must converge to zero. Interpreting the distance $\overline{b} - b$

[22] If condition (19) of Section 3.4 is not satisfied, the equilibrium tax rate is greater than τ_θ^*. If condition (19) is satisfied, matters depend on the revenue requirement. For sufficiently high revenue requirements, the equilibrium tax rate in the low productivity state must again be greater than τ_L^*. This is because as r_L approaches $r_L(\overline{b})$, the equilibrium tax rate approaches the revenue maximizing level 1/2, which exceeds τ_L^*. However, in either state for sufficiently low revenue requirements, the equilibrium tax rate can be less than τ_θ^*.

[23] Papers trying to measure the multiplier impacts of different policies include Alesina and Ardagna (2010), Barro and Redlick (2011), Blanchard and Perotti (2002), Mountford and Uhlig (2009), Nakamura and Steinsson (2011), Ramey (2011a), Romer and Romer (2010), Serrato and Wingender (2011), and Shoag (2010). A central issue in this literature is the relative size of tax cut and public spending multipliers. For overviews and discussion of the literature see Auerbach, Gale, and Harris (2010), Parker (2011), and Ramey (2011b).

[24] The importance of non-linearities and the difficulties this creates for measurement is a theme of Parker (2011).

as the economy's *fiscal space*, this result is simply saying that when the economy's fiscal space becomes very small (as a result, say, of a sequence of negative shocks or less inclusive political decision-making), its efforts to fight further negative shocks with fiscal policy will necessarily be limited.[25] We conjecture that, more generally, the magnitude of the stimulus as measured by $\rho b - r_L(b)$ will depend negatively on the initial debt level b. We also expect that as a result of this, an economy will experience higher increases in unemployment as a result of negative shocks when it has a higher debt level. This in turn suggests that employment levels in an economy will be more volatile when that economy is more indebted. We will return to this idea in Section 4.

3.4 Political distortions

There are two types of distortions that can arise in our economy. The first is unemployment: some of the available workforce is not utilized. The second is an inefficient output mix: the workforce that is utilized is not allocated optimally between private and public production. If policies are chosen by a planner seeking to maximize aggregate societal utility, it can be shown that there will be no distortions in the long run.[26] The way in which the government achieves this first best outcome is by accumulating bond holdings. In the long run, in every period the government hires sufficient public sector workers to provide the Samuelson level of the public good and sets taxes so that the private sector has the incentive to hire the remaining workers. If these taxes are sufficiently low that tax revenues fall short of the costs of public good provision, the earnings from government bond holdings are used to finance the shortfall. Surplus bond earnings are rebated back to citizens via a uniform transfer. This result parallels similar results for the tax smoothing model (Aiyagari et al 2002, Battaglini and Coate 2008, and Barshegyan, Battaglini, and Coate 2013).[27]

As we have already seen, in political equilibrium, there is always unemployment under our

[25] For more on the concept of fiscal space and an attempt to measure it see Ostroy, Ghosh, Kim and Qureshi (2010).

[26] An extensive analysis of the benevolent government solution can be found in our NBER working paper, Battaglini and Coate (2011).

[27] In the tax smoothing model, the government eventually accumulates sufficient assets so that it can finance government spending needs at first best levels without distortionary taxation. Thus, there are no distortions in the long run. When the need for revenue is low, the government not only pays down the debt that was issued in times of high revenue need, it also reduces the base debt level. Gradually, over time, it starts to accumulate a stock of assets. It only stops accumulating when the interest earnings from assets are sufficient to completely eliminate the need for distortionary taxation. While the nature of the distortions are very different in this model, the same forces are operative.

assumptions. It should also be noted that the output mix will be distorted conditional on the unemployment level. This means that either the public sector is too large or too small. The direction of the distortion turns out to depend on the underlying parameters of the economy in a relatively simple way. In the Appendix, we show that with unemployment rate u the output mix is distorted towards the private good when

$$n_e > \frac{1 - u - 2\gamma/A_\theta}{1 - u + A_\theta/2\xi}. \tag{19}$$

Otherwise, it is distorted towards public production. Condition (19) is more likely to hold the larger is the number of entrepreneurs n_e, the larger is the economy's preference for public goods γ, and the larger is the unemployment rate u.

To gain intuition for this result, recall that the first best output mix for any given unemployment level u can be found by solving problem (15) with $q = 1$, no revenue requirement, and the resource constraint with n_w replaced by the number of workers actually employed $n_w(1 - u)$. In the solution, the government chooses the Samuelson level of the public good and adjusts the tax rate to get the private sector to employ the remaining available workers. Relative to this problem, the mwc puts more weight on raising revenue (either because it wants revenues for transfers or because it needs to meet the revenue requirement). Thus, relative to the first best, the mwc is choosing tax rates and public production that keep employment constant but generate more revenue. Keeping employment constant requires that if taxes are raised, any private sector workers laid off are employed in the public sector. Conversely, if public production is reduced, entrepreneurs must be incentivized to hire the displaced public sector workers. Clearly, if entrepreneurs can be induced to hire more workers for only a very small tax cut, then it makes sense to reduce public production. The savings from reducing public production will exceed the loss in tax revenues. The employment response for any given tax cut will be greater, the higher are the first best taxes. Accordingly, when first best taxes are high, reducing public production will be the optimal way to distort the output mix. First best taxes will be high when the first best public good level is high (high γ), when the size of the private sector is large (high n_e), and when the unemployment rate is large (high u).[28]

[28] These assertions can be verified from the formula for first best taxes developed in the Appendix.

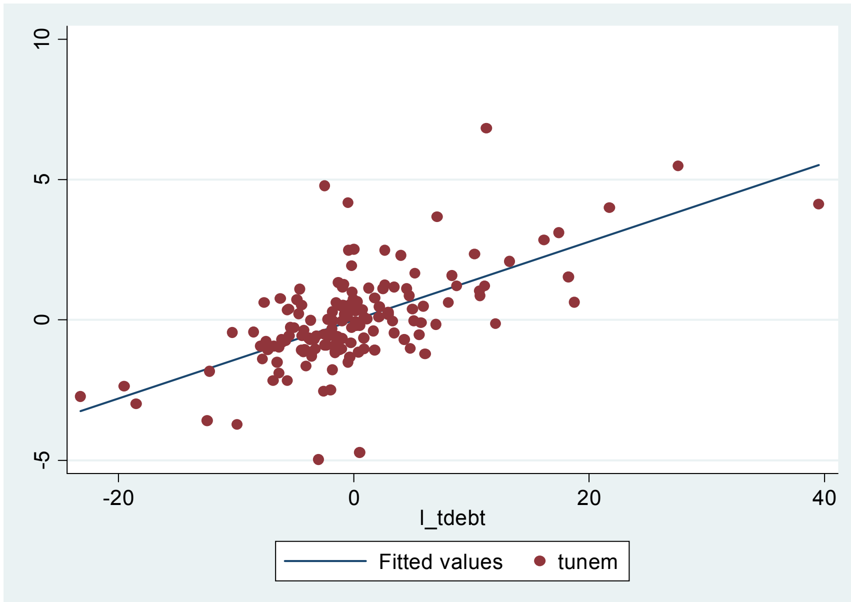

Figure 5:

4 Empirical implications and some evidence

Our theory has two unambiguous qualitative implications. The first is that the dynamic pattern of debt is counter-cyclical. More precisely, increases in debt should be positively correlated with reductions in output and visa versa. This follows from Proposition 2. This implication also emerges from tax smoothing models and simple Keynesian theories of fiscal policy, so there is nothing particularly distinctive about it. Empirical support for this prediction for U.S. debt is provided by Barro (1986).

The second implication is that, ceteris paribus, the larger an economy's pre-existing debt level, the higher will be its unemployment rate. This follows from Proposition 3. Since we are not aware of any other theoretical work that links pre-existing debt and unemployment, we believe this is a novel prediction. Assessing its validity is not immediate because the empirical literature does not appear to have extensively analyzed the relationship. A positive correlation between pre-existing debt and unemployment has been noted by Bertola (2011) using a panel of OECD countries from 1980 up to 2003.[29] In Figure 5 and Table 1 we have augmented Bertola's analysis considering

[29] A positive correlation between debt and unemployment is also found by Fedeli and Forte (2011) and Fedeli,

panel data from 2006 to 2010 and controlling for economically relevant variables.[30] Each point in Figure 5 corresponds to an OECD country in a given year in the five year period 2006 to 2010.[31] The height of a point on the vertical axis measures the country's unemployment rate in that year less its average rate over the five year period. The length of a point on the horizontal axis measures the country's debt/GDP ratio at the beginning of the prior year less its average ratio.[32] The Figure reveals a strong positive correlation. In Table 1, the level of unemployment in period t is regressed on the level of debt at the beginning of period $t-1$ and a selection of controls.[33] Country and year fixed effects are included. Column 1 presents the basic results. Column 2 controls for interest rates and Column 3 controls for both interest rates and for non linear effects including a variable equal to the debt/GDP ratio if it is larger than 90%. As shown, in each specification, the effect of debt on unemployment is positive and highly significant. The variable for the 90% threshold is not significant.

A further idea concerning the relationship between debt and unemployment suggested by the theory is that unemployment in a country might be more responsive to shocks when that country has higher debt. As discussed in Section 3.3, the logic of the model suggests that, with lower debt, the country will be able to better self insure against shocks. This in turn suggests that any given negative shock is likely to result in a bigger bump up in unemployment. Some preliminary support for this idea is presented in columns 4-6 of Table 1 that document a positive and significant correlation between the absolute value of the change in unemployment rate between period t and period $t-1$ in a country on the level of debt at the beginning of period $t-1$.

A final point worth noting is that the model has no robust implications for the cyclical behavior of taxes and public spending.[34] Depending on the parameters, when the economy experiences

Forte, and Ricchi (2012) in a cointegration analysis of OECD data from 1970 to 2009.

[30] Our focus on a short panel is motivated by the desire to avoid non-stationarity problems along the time dimension.

[31] The panel includes 30 countries. Turkey, Mexico, Luxembourg and Estonia were dropped from the regression because values of the debt/GDP variable for the relevant years are missing in the OECD database.

[32] We use the debt level at the beginning of period $t-1$ to deal with the objection that the debt level at the beginning of period t may already reflect stimulus efforts designed to deal with headwinds in the economy which foretell higher unemployment in period t. In fact, almost identical results arise when we use the debt level at the beginning of period t.

[33] In Table 1 the variable *debt* is the debt/GDP ratio, *debt_plus_90* is a variable equal to *debt* if *debt* is larger than 90%, *l_dependency* is the % of working age population, *l_popgrowth* is the annual % rate of population growth, *l_open* measures imports plus exports in goods and services as a % of GDP, *l_bonds* is the interest rate on 10 year bonds.

[34] There is an extensive literature on the cyclical behavior of public spending and taxes. See, for example, Alesina, Campante, and Tabellini (2008), Barro (1986), Barshegyan, Battaglini, and Coate (2013), Furceri and

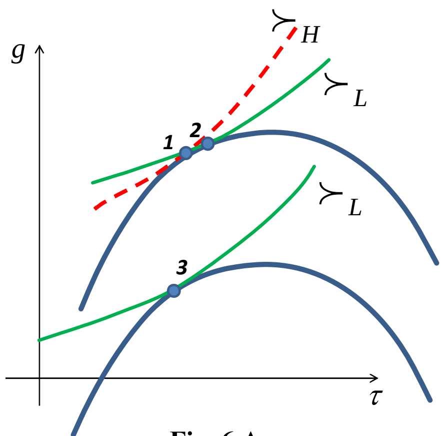

Fig. 6.A

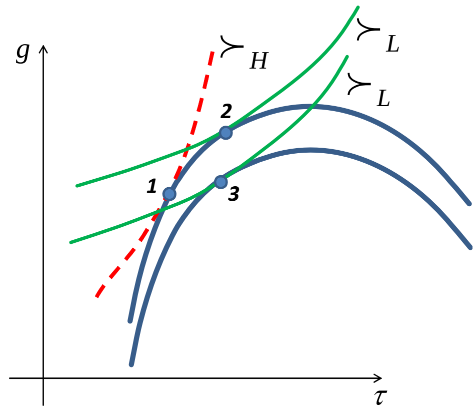

Fig. 6.B

Figure 6:

a negative shock, public spending could increase or decrease, and tax rates could increase or decrease. Two effects are at work. First, when private sector productivity decreases the mwc's indifference curve becomes flatter, so if the budget line did not change, τ and g would increase (this is represented by the move from point 1 to point 2 in Figure 6). Intuitively, the marginal cost of raising taxes is lower because the private sector is less productive and therefore taxation results in a lower output response. It therefore becomes optimal to increase the size of the public sector. The reduction in private sector productivity, however, does impact the budget line. Specifically, it both shifts downward and becomes flatter. Intuitively, any given tax raises less revenue and any given increase in taxes results in a smaller revenue increase. Although the downward shift is partially compensated by an increase in debt, the combination of the downward shift and the flattening makes the net effect on taxes and public spending ambiguous. This is illustrated in Figure 6. When the economy experiences a positive shock, taxes and public spending move from point 1 to point 3. In Fig. 6.A, public spending and taxes decrease and in Fig. 6.B, public spending and taxes increase.

Karras (2011), Gavin and Perotti (1997), Lane (2003), and Talvi and Vegh (2005). In light of the variety of empirical correlations found in the literature, the fact that the model predicts no clear pattern of behavior is perhaps a virtue.

5 Conclusion

This paper has presented a political economy theory of the interaction between fiscal policy and unemployment. Under our assumptions, the economy will always have unemployment. This unemployment will be higher when the private sector experiences negative shocks. To mitigate this additional unemployment, the government will employ debt-financed fiscal stimulus plans, which will involve both tax cuts and public production increases. When the private sector is healthy, the government will contract debt until it reaches a floor level. Unemployment levels are weakly increasing in the economy's debt level, strictly so when the private sector experiences negative shocks. Conditional on the level of workers employed, the mix of public and private output is distorted.

There are many different directions in which the ideas presented here might usefully be developed. In terms of the basic model, it would be desirable to incorporate a richer model of unemployment into the analysis. The search theoretic approach of Michaillat (2012) would seem promising in this regard since it allows for both rationing unemployment (as in this paper) and frictional unemployment. This would permit us to move beyond the sharp distinction between full employment and unemployment. With respect to political decision-making, it would be interesting to introduce class conflict into the analysis. The current model limits the conflict among citizens to disagreements concerning the allocation of transfers between districts. This is made possible by assuming that each legislator behaves so as to maximize the aggregate utility of the citizens in his district. Alternatively, we could assume that legislators either represent workers or entrepreneurs in their districts. This would introduce an additional conflict over policies in the sense that workers prefer policies that keep wages and employment high, while entrepreneurs prefer policies which keep profits high. Such class conflict may have important implications for the choice of fiscal policy. Finally, it would be interesting to introduce money into the model and explore how monetary policy interacts with fiscal policy and unemployment. Comparing the control of monetary policy by legislators and a central bank would be of particular interest.

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6 Appendix

6.1 Proof of Proposition 1

As argued in the text, Problem (14) is equivalent to Problem (15). The Lagrangian for Problem (15) is

$$L = x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + (\lambda + q - 1)\left(R_\theta(\tau, \underline{\omega}) - \underline{\omega} g - r\right) + \mu \left(n_w - g - \frac{x_\theta(\tau)}{A_\theta}\right).$$

Thus, λ is the multiplier on the budget constraint and μ is the multiplier on the resource constraint. Using the expressions for $x_\theta(\tau)$ and $R_\theta(\tau, \underline{\omega})$ in (8) and (9), the first order conditions with respect to g and τ are

$$\frac{\gamma}{g} = (\lambda + q - 1)\underline{\omega} + \mu, \tag{20}$$

and

$$(\lambda + q - 1)(1 - 2\tau)(A_\theta - \underline{\omega}) = \tau A_\theta + (1 - \tau)\underline{\omega} - \mu. \tag{21}$$

We begin by characterizing the optimal policies for the mwc ignoring the budget constraint (the unconstrained optimal policies) which we have denoted $(\tau_\theta^q, g_\theta^q)$. These policies can be obtained from the first order conditions by setting λ equal to zero. There are two possibilities depending on whether the resource constraint binds. If μ equals zero, (20) and (21) imply that the solution is given by

$$(\tau_\theta^q, g_\theta^q) = \left(\frac{(q-1)A_\theta - q\underline{\omega}}{(A_\theta - \underline{\omega})(2q-1)}, \frac{\gamma}{(q-1)\underline{\omega}}\right). \tag{22}$$

It follows that the resource constraint does not bind if at these values of $(\tau_\theta^q, g_\theta^q)$, it is the case that $g_\theta^q + \frac{x_\theta(\tau_\theta^q)}{A}$ is less than n_w. This condition amounts to

$$n_e\left[\frac{q(A_\theta - \underline{w}) + \underline{w}}{\xi(2q-1)}\right] + \frac{\gamma}{(q-1)\underline{w}} < n_w, \tag{23}$$

which is implied by Assumption 1. We conclude that the policies $(\tau_\theta^q, g_\theta^q)$ are given by (22).

Now suppose that the revenue requirement r is less than or equal to r_θ^q. Recall that by definition, $r_\theta^q = R_\theta(\tau_\theta^q, \underline{\omega}) - \underline{\omega} g_\theta^q$. If $r \leq r_\theta^q$, the budget constraint will not be binding and the optimal policies will be $(\tau_\theta^q, g_\theta^q)$ as given in (22). The level of transfers will be $r_\theta^q - r$. Given Assumption 1, there will be unemployment at these policies.

Now suppose the revenue requirement r exceeds r_θ^q. When $r > r_\theta^q$, the budget constraint must bind. The resource constraint will continue not to bind because the budget constraint must lie strictly below the full employment line (see Fig. 3.B). Thus, $\lambda > 0$ and $\mu = 0$. Substituting (20) into (21), we obtain

$$g = \frac{\gamma(1-2\tau)(A_\theta - \underline{\omega})}{\underline{\omega}\left(\tau(A_\theta - \underline{\omega}) + \underline{\omega}\right)}. \tag{24}$$

Substituting (24) into the budget constraint, we obtain:

$$\tau n_e(1-\tau)(A_\theta - \underline{\omega})^2/\xi - \left(\frac{\gamma(1-2\tau)(A_\theta - \underline{\omega})}{\tau(A_\theta - \underline{\omega}) + \underline{\omega}}\right) = r \tag{25}$$

This equation has a unique solution $\widehat{\tau}_\theta(r)$ in the relevant range for τ, i.e. $[0, 1/2]$. Since the right hand side of (25) is always increasing for τ less than $1/2$, $\widehat{\tau}_\theta(r)$ is increasing in r. The associated value of g, $\widehat{g}_\theta(r)$, is obtained from (24). Since the right hand side of (24) is decreasing in τ, $\widehat{g}_\theta(r)$ is decreasing in r. Furthermore, note that unemployment is an increasing function of τ and a decreasing function of g, so it is increasing in r as well. ■

6.2 Proof of Proposition 2

The proof is broken into three parts. In Subsection 6.2.1 we characterize b^q - the lower bound of the equilibrium debt distribution. In Subsection 6.2.2 we prove that debt behaves in a counter-cyclical way. In Subsection 6.2.3 we prove that a non-degenerate stable distribution exists and has full support in $\left[b^q, \overline{b}\right]$.

6.2.1 The lower bound b^q

Consider problem (12). When the budget constraint is not binding, the mwc will choose a debt level from the set

$$\mathcal{X} = \arg\max_{b' \leq \overline{b}} \left\{ qb' + \beta E V_{\theta'}(b') \right\}.$$

Our first result is:

Lemma A.1. *In any equilibrium, the set $\mathcal{X}$ is a singleton.*

Proof. This proof is relegated to the on-line Appendix.

Given Lemma A.1, we define b^q to be the unique element of the set $\mathcal{X}$. We also define b^q_θ to be the value of debt such that the triple $(\tau^q_\theta, g^q_\theta, b^q)$ satisfies the constraint that $B_\theta(\tau^q_\theta, g^q_\theta, b^q, b^q_\theta, \underline{\omega})$ equal 0. This is given by:

$$b^q_\theta = \frac{r^q_\theta + b^q}{1+\rho}. \tag{26}$$

Then, if the debt level b is such that $b \leq b_\theta^q$ the tax-public good-debt triple is $(\tau_\theta^q, g_\theta^q, b^q)$ and the mwc uses the budget surplus $B_\theta(\tau_\theta^q, g_\theta^q, b^q, b_\theta^q, \underline{\omega})$ to finance transfers. If $b > b_\theta^q$ the budget constraint binds so that no transfers are given. Tax revenues net of public good costs strictly exceed r_θ^q and the debt level strictly exceeds b^q. In this case, the policies solve the problem

$$\max_{(\tau,g,b')} \left\{ \begin{array}{c} b' - (1+\rho)b + x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + \beta E V_{\theta'}(b') \\ s.t.\ B_\theta(\tau, g, b', b, \underline{\omega}) \geq 0 \ \&\ b \leq \overline{b} \end{array} \right\}. \tag{27}$$

Note also that, since r_H^q exceeds r_L^q, it must be the case that $b_H^q > b_L^q$.

Further information on the debt level b^q can be obtained by using a first order condition to characterize it. However, before we can do this, we must first establish that the value function is differentiable. We have:

Lemma A.2. *The equilibrium value function $V_\theta(b)$ is differentiable for all $b \neq b_\theta^q$. Moreover,*

$$-\beta V_\theta'(b) = \left\{ \begin{array}{cc} 1 & if\ b < b_\theta^q \\ 1 + \frac{\tau_\theta(b)A_\theta + (1-\tau_\theta(b))\underline{\omega}}{(1-2\tau_\theta(b))(A_\theta - \underline{\omega})} & if\ b > b_\theta^q \end{array} \right. .$$

Proof. This proof is relegated to the on-line Appendix.

We can now show:

Lemma A.3. $b^q \in [b_L^q, b_H^q]$**.**

Proof. From the definition of b^q, we know that if V_L and V_H are differentiable at b^q it must be the case that

$$q = -\beta E V_\theta'(b^q) \tag{28}$$

Assume first that $b^q < b_L^q$. Then, by Lemma A.2, (28) would imply $q = 1$, a contradiction. Assume next that $b^q > b_H^q$. This would imply that for each state θ, we have that $\tau_\theta(b^q) > \tau_\theta^q$. Using the first order conditions for τ_θ^q and the expressions in Lemma A.2, we can show that this implies $\beta V_\theta'(b^q) < -q$. This implies: $-\beta E V_\theta'(b^q) > q$: again, a contradiction. We conclude that $b^q \in [b_L^q, b_H^q]$ as claimed. ■

We can use this result to establish the assertion in the proposition that $b^q > r_L^q/\rho$. We have from Lemma A.3 that $b^q \geq b_L^q = \frac{r_L^q + b^q}{1+\rho}$. Multiplying this inequality through by $1+\rho$ yields the result.

6.2.2 Proof of countercyclical behavior

We begin with the following useful result.

Lemma A.4. *For all $b \in [b_L^q, \bar{b}]$ it is the case that $\lambda_L(b) > \lambda_H(b)$, where $\lambda_\theta(b)$ is the Lagrange multiplier on the budget constraint for the problem (27).*

Proof. Let $b \in [b_L^q, \bar{b}]$. Suppose, contrary to the claim, that $\lambda_H(b) \geq \lambda_L(b)$. Then, by the concavity of the value function, we know that in the solution to problem (27) $b'_H(b) \geq b'_L(b)$. Moreover, we claim that:

$$R_H(\tau_H(b), \underline{\omega}) - \underline{\omega} g_H(b) > R_L(\tau_L(b), \underline{\omega}) - \underline{\omega} g_L(b), \tag{29}$$

where $(\tau_\theta(b), g_\theta(b))$ denote the solutions to problem (27). From the first order conditions for problem (27), we know that for each state θ, $\underline{\omega}\lambda_\theta(b) = \frac{\gamma}{g_\theta(b)}$ and that

$$\lambda_\theta(b) = \frac{\underline{\omega} + \tau_\theta(b)(A_\theta - \underline{\omega})}{(1 - 2\tau_\theta(b))(A_\theta - \underline{\omega})}.$$

From the first condition, we conclude that $g_H(b) \leq g_L(b)$. It follows that to establish the result it suffices to show that $R_L(\tau_L(b), \underline{\omega})$ is less than $R_H(\tau_H(b), \underline{\omega})$. To this end, note that the second condition implies that

$$\tau_\theta(b) = \frac{\lambda_\theta(b)(A_\theta - \underline{\omega}) - \underline{\omega}}{(1 + 2\lambda_\theta(b))(A_\theta - \underline{\omega})}.$$

Define the function

$$\varphi(A, \lambda) = \frac{\lambda(A - \underline{\omega}) - \underline{\omega}}{(1 + 2\lambda)(A - \underline{\omega})}$$

Note that

$$\begin{aligned}
\frac{\partial\varphi(A, \lambda)}{\partial A} &= \frac{(1 + 2\lambda)(A - \underline{\omega})\lambda_\theta(b) - (1 + 2\lambda)\left[\lambda(A - \underline{\omega}) - \underline{\omega}\right]}{(1 + 2\lambda)^2(A - \underline{\omega})^2} \\
&= \frac{(1 + 2\lambda)\underline{\omega}}{(1 + 2\lambda)^2(A - \underline{\omega})^2} > 0
\end{aligned}$$

and that

$$\frac{\partial\varphi(A, \lambda)}{\partial\lambda} = \frac{(1 + 2\lambda)(A - \underline{\omega})(A - \underline{\omega}) - 2(A - \underline{\omega})\left[\lambda(A - \underline{\omega}) - \underline{\omega}\right]}{(1 + 2\lambda)^2(A - \underline{\omega})^2} = \frac{A + \underline{\omega}}{(1 + 2\lambda)^2(A - \underline{\omega})} > 0$$

Thus, we have that $\tau_H(b) = \varphi(A_H, \lambda_H(b)) > \varphi(A_L, \lambda_L(b)) = \tau_L(b)$. Since $\tau_H(b) < 1/2$, this inequality implies that $R_H(\tau_H(b), \underline{\omega}) > R_L(\tau_L(b), \underline{\omega})$. Since $b \geq b_L^q$, moreover, we know that

$\lambda_L(b) > 0$ and hence that the budget constraint is binding in low productivity state. Thus, if (29) holds, we have

$$R_H(\tau_H(b), \underline{\omega}) - \underline{\omega} g_H(b) + b'_H(b) - (1+\rho)b > R_L(\tau_L(b), \underline{\omega}) - \underline{\omega} g_L(b) + b'_L(b) - (1+\rho)b = 0.$$

This implies $\lambda_H(b) = 0$. So we have $\lambda_H(b) < \lambda_L(b)$, a contradiction. ■

We now prove:

Lemma A.5. *In equilibrium: (i) $b'_L(b) > b$ for all $b < \overline{b}$, and, (ii) $b'_H(b) > b$ for all $b \leq b^q$ and $b'_H(b) < b$ for all $b \in (b^q, \overline{b}]$.*

Proof (i) We need to show that $b'_L(b) > b$ for all $b < \overline{b}$. Let $b < \overline{b}$. Suppose first that $b \leq b^q_L$. Then, we have that $b'_L(b) = b^q \geq b^q_L > b$. Suppose next that $b > b^q_L$. We know that $b'_L(b) > b^q$ and that $b'_L(b)$ satisfies the first order condition:

$$1 + \lambda_L(b) \geq -\beta E V'_\theta(b'_L(b)) \; (= \text{ if } b'_L(b) < \overline{b})$$

where $\lambda_L(b)$ is the Lagrangian multiplier on the budget constraint on the maximization problem (27). We also know from the proof of Lemma A.2 that

$$-\beta V'_\theta(b) = \begin{cases} 1 + \lambda_\theta(b) & \text{if } \; b > b^q_\theta \\ 1 & \text{if } \; b < b^q_\theta \end{cases} \tag{30}$$

Suppose that $b'_L(b) \leq b$. Then if $b \geq b^q_H$, we have that

$$1 + \lambda_L(b) = -\beta E V'_\theta(b'_L(b)) \leq -\beta E V'_\theta(b) = (1-\alpha)(1+\lambda_L(b)) + \alpha(1+\lambda_H(b)) < 1 + \lambda_L(b)$$

since $\lambda_L(b) > \lambda_H(b)$ for all $b \geq b^q_L$ by Lemma A.4. If $b < b^q_H$, we have that

$$1 + \lambda_L(b) = -\beta E V'_\theta(b'_L(b)) \leq -\beta E V'_\theta(b) = (1-\alpha)(1+\lambda_L(b)) + \alpha < 1 + \lambda_L(b).$$

(ii) We first show that $b'_H(b) > b$ for all $b \leq b^q$. Let $b \leq b^q$. Then since $b^q < b^q_H$, we know that $b'_H(b) = b^q > b$. We next show that $b'_H(b) < b$ for all $b \in (b^q, \overline{b}]$. Let $b \in (b^q, \overline{b}]$. Suppose first that $b \leq b^q_H$. Then we know that $b'_H(b) = b^q < b$. Now suppose that $b > b^q_H$. We know that $b'_H(b)$ satisfies the first order condition:

$$1 + \lambda_H(b) \geq -\beta E V'_\theta(b'_H(b)) \;\; (= \text{ if } b'_H(b) < \overline{b})$$

Suppose that $b'_H(b) \geq b$. Then since $b > b_H^q$ we have that

$$1 + \lambda_H(b) \geq -\beta E V'_\theta(b'_H(b)) \geq -\beta E V'_\theta(b) = (1-\alpha)(1 + \lambda_L(b)) + \alpha(1 + \lambda_H(b)) > 1 + \lambda_H(b),$$

where the last step relies on (30) and the fact that by Lemma A.4 $\lambda_L(b) > \lambda_H(b)$. This is a contradiction. ■

6.2.3 The stable distribution

Let $\psi_t(b)$ denote the distribution function of the current level of debt at the beginning of period t. The distribution function $\psi_0(b)$ is exogenous and is determined by the economy's initial level of debt b_0. The transition function implied by the equilibrium is given by

$$H(b, b') = \begin{cases} \Pr\{\theta' \text{ s.t. } b'_{\theta'}(b) \leq b'\} & \text{if } \exists\ \theta' \ \ s.t.\ b'_{\theta'}(b) \leq b' \\ 0 & otherwise \end{cases},$$

for any $b' \in [b^q, \overline{b}]$. $H(b, b')$ is the probability that in the next period the initial level of debt will be less than or equal to $b' \in [b^q, \overline{b}]$ if the current level of debt is b. Using this notation, the distribution of debt at the beginning of any period $t \geq 1$ is defined inductively by $\psi_t(b) = \int_z H(z, b) d\psi_{t-1}(z)$. The sequence of distributions $\langle \psi_t(b) \rangle$ converges to the distribution $\psi(b)$ if we have that $\lim_{t\to\infty} \psi_t(b) = \psi(b)$ for all $b' \in [b^q, \overline{b}]$. Moreover, $\psi^*(b)$ is an invariant distribution if

$$\psi^*(b) = \int_z H(z, b) d\psi^*(z).$$

We now establish that any sequence of equilibrium debt distributions $\langle \psi_t(b) \rangle$ converges to a unique invariant distribution $\psi^*(b)$.

It is easy to prove that the transition function $H(b, b')$ has the Feller Property and that it is monotonic in b (see Ch. 8.1 in Stokey, Lucas and Prescott (1989) for definitions). Define the function $H^m(b, b')$ inductively by $H^0(b, b') = H(b, b')$ and $H^m(b, b') = \int_z H(z, b') dH^{m-1}(b, z)$. By Theorem 12.12 in Stokey, Lucas and Prescott (1989), therefore, the result follows if the following "mixing condition" is satisfied:

Mixing Condition: *There exists an* $\epsilon > 0$ *and* $m \geq 0$, *such that* $H^m(\overline{b}, b^q) \geq \epsilon$ *and* $H^m(b^q, b^q) \leq 1 - \epsilon$.

The proof that this condition is satisfied can be found in the on-line Appendix.

To prove that the stable distribution has full support in $[b^q, \overline{b}]$ we show that for any $b \in [b^q, \overline{b}]$, $\psi^*(b) \in (0, 1)$. The details can be found in the on-line Appendix. ■

6.3 Proof of Proposition 3

The fact that there is always unemployment follows immediately from Proposition 1. To show that, for any given debt level, unemployment is higher when private sector productivity is low than when it is high, we need to show that for all b in the support of the invariant distribution, we have that

$$g_H(b) + n_e(1 - \tau_H(b))(A_H - \underline{\omega})/\xi > g_L(b) + n_e(1 - \tau_L(b))(A_L - \underline{\omega})/\xi.$$

Suppose first that $b \in [b^q, b_H^q]$ where b_H^q was defined in the proof of Proposition 2. In this case

$$(\tau_H(b), g_H(b)) = (\tau_H^q, g_H^q) = \left(\frac{(q-1)\, A_H - q\underline{\omega}}{(A_H - \underline{\omega})\,(2q-1)}, \frac{\gamma}{(q-1)\,\underline{\omega}} \right),$$

so that $1 - \tau_H(b) = \frac{qA_H - (q-1)\underline{\omega}}{(A_H - \underline{\omega})(2q-1)}$ and

$$g_H(b) + n_e(1 - \tau_H(b))(A_H - \underline{\omega})/\xi = \frac{\gamma}{(q-1)\,\underline{\omega}} + \frac{n_e\,(qA_H - (q-1)\,\underline{\omega})}{\xi\,(2q-1)}.$$

On the other hand, from the proof of Proposition 2, we know that $b > b_L^q$, which implies that $\tau_L(b) > \tau_L^q$ and $g_L(b) < g_L^q$. This implies that

$$\begin{aligned} g_L(b) + n_e(1 - \tau_L(b))(A_L - \underline{\omega})/\xi &< \frac{\gamma}{(q-1)\,\underline{\omega}} + \frac{n_e\,(qA_L - (q-1)\,\underline{\omega})}{\xi\,(2q-1)} \\ &< \frac{\gamma}{(q-1)\,\underline{\omega}} + \frac{n_e\,(qA_H - (q-1)\,\underline{\omega})}{\xi\,(2q-1)}. \end{aligned}$$

Next suppose that $b \in (b_H^q, \overline{b})$. In this case, as argued in the proof of Proposition 2, the policies in each state θ satisfy the first order conditions for the problem (27) $\frac{\gamma}{g_\theta(b)} = \lambda_\theta(b)\underline{\omega}$, and $\lambda_\theta(b)(1 - 2\tau)(A_\theta - \underline{\omega}) = \tau A_\theta + (1-\tau)\underline{\omega}$, where $\lambda_\theta(b)$ is the Lagrange multiplier on the budget constraint. Thus, $g_\theta(b) = \frac{\gamma}{\lambda_\theta(b)\underline{\omega}}$, and $1 - \tau_\theta(b) = \frac{(A_\theta - \underline{\omega})(\lambda_\theta(b)+1) + \underline{\omega}}{(A_\theta - \underline{\omega})(1 + 2\lambda_\theta(b))}$. This means that

$$g_\theta(b) + n_e(1 - \tau_\theta(b))(A_\theta - \underline{\omega})/\xi = \frac{\gamma}{\lambda_\theta(b)\underline{\omega}} + \frac{n_e\,((\lambda_\theta(b) + 1)\,A_\theta - \lambda_\theta(b)\underline{\omega})}{\xi\,[2\lambda_\theta(b) + 1]}.$$

We know from Lemma A.4 that $\lambda_H(b) < \lambda_L(b)$. This implies that

$$\frac{\gamma}{(\lambda_L(b) + q - 1)\underline{\omega}} < \frac{\gamma}{(\lambda_H(b) + q - 1)\underline{\omega}},$$

which means that public employment is lower in the low productivity state. Thus, we just need to show that private employment is lower as well. Defining the function $h_\theta(\lambda) = \frac{(\lambda+1)A_\theta - \lambda\underline{\omega}}{2\lambda+1}$, it is enough to show that $h_H(\lambda_H(b)) > h_L(\lambda_L(b))$. Note that

$$h_L'(\lambda) = \frac{(2\lambda+1)\,(A_L - \underline{\omega}) - 2\,((\lambda+1)\,A_L - \lambda\underline{\omega})}{(2\lambda+1)^2} = \frac{-\,(A_L + \underline{\omega})}{(2\lambda+1)^2} < 0$$

It follows that $h_L(\lambda_H(b)) > h_L(\lambda_L(b))$. In addition, it is clear that $h_H(\lambda_H(b)) > h_L(\lambda_H(b))$.

For the remainder of the proposition, it suffices to establish the properties asserted to be true in the paragraph preceding the statement of the proposition. Thus, we need to establish the following properties: (i) that $r_\theta(b)$ is increasing in b for each state θ, and (ii) that $r_L(b^q) > r_L^q$, and $r_H(b^q) \leq r_H^q$. For property (i), assume first that $b \leq b_\theta^q$ where b_θ^q was defined in the proof of Proposition 2. In this case $b'_\theta(b) = b^q$, so $r_\theta(b) = (1+\rho)\,b - b'_\theta(b)$ is increasing in b. Assume now that $b > b_\theta^q$. Then, as argued in the proof of Proposition 2, we know that $B_\theta(\tau_\theta(b), g_\theta(b), b'_\theta(b), b, \underline{\omega}) = 0$, implying that $b(1+\rho) - b'_\theta(b) = R_\theta(\tau_\theta(b), \underline{\omega}) - \underline{\omega} g_\theta(b)$. An increase in b implies that $\lambda_\theta(b)$ increases, implying that $R_\theta(\tau_\theta(b), \underline{\omega}) - \underline{\omega} g_\theta(b)$ increases in b. Property (ii) follows from Lemma A.3 and the definition of b_θ^q in (26). ■

6.4 Proof of the claim in Section 3.4

In Section 3.4, we asserted that with unemployment rate u the output mix is distorted towards the private good when

$$n_e > \frac{1 - u - 2\gamma/A_\theta}{1 - u + A_\theta/2\xi}. \tag{31}$$

Otherwise, it is distorted towards public production. With unemployment rate u, the first best policies in state θ solve the problem

$$\max_{(\tau,g)} \left\{ \begin{array}{c} x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g \\ s.t.\ g + \frac{x_\theta(\tau)}{A_\theta} \leq n_w(1-u) \end{array} \right\}.$$

The first order conditions with respect to g and τ imply $\tau A_\theta + (1-\tau)\underline{\omega} = \frac{\gamma}{g}$. Combining this equation with the unemployment-modified resource constraint and solving we find that the first best policies satisfy

$$\left(\tau_\theta^1(u), g_\theta^1(u)\right) = \left(\begin{array}{c} 1 - \frac{\xi\left(n_w(1-u) - g_\theta^1(u)\right)}{n_e(A_\theta - \underline{\omega})}, \\ \frac{\sqrt{(A_\theta n_e - \xi n_w(1-u))^2 + 4\xi n_e \gamma} - (A_\theta n_e - \xi n_w(1-u))}{2\xi} \end{array} \right). \tag{32}$$

As argued in the text, relative to the first best policies, the equilibrium policies will be distorted in the direction of raising more revenue. The policies that maximize the revenue raised at this unemployment rate lie where the unemployment-modified resource constraint is tangent to the budget line. Given (18), this tangency occurs at the policies $(\tau_\theta^*, g_\theta^*(u))$ satisfying the equations

$$\frac{\partial R_\theta(\tau_\theta^*, \underline{\omega})/\partial\tau}{\underline{\omega}} = \frac{n_e(A_\theta - \underline{\omega})}{\xi}, \tag{33}$$

and

$$g_\theta^*(u) = n_w(1-u) - \frac{x_\theta(\tau_\theta^*)}{A_\theta}. \tag{34}$$

The first equation ensures that the slope of the unemployment-modified resource constraint is equal to that of the budget line. The second equation implies that the tangency occurs at the appropriate unemployment level.

The equilibrium policies will lie on the unemployment-modified resource constraint somewhere between $\left(\tau_\theta^1(u), g_\theta^1(u)\right)$ and $(\tau_\theta^*, g_\theta^*(u))$. This is because the equilibrium policies are distorted in the direction of raising more revenue and moving towards $(\tau_\theta^*, g_\theta^*(u))$ raises more revenue. Thus, if $\tau_\theta^1(u)$ is less than τ_θ^*, the equilibrium output mix is distorted towards public production and if $\tau_\theta^1(u)$ is greater than τ_θ^*, the output mix is distorted towards the private good. Using (33) we have that $\tau_\theta^* = \frac{(A_\theta - 2\underline{\omega})}{2(A_\theta - \underline{\omega})}$. From (32), we have that $\tau_\theta^1(u) = 1 - \frac{\xi\left(n_w(1-u) - g_\theta^1(u)\right)}{n_e(A_\theta - \underline{\omega})}$. Thus,

$$\tau_\theta^1(u) \lesseqgtr \tau_\theta^* \Leftrightarrow g_\theta^1(u) \lesseqgtr \frac{2\xi n_w(1-u) - n_e A_\theta}{2\xi}.$$

Using the expression for $g_\theta^1(u)$ in (32), we have that

$$g_\theta^1(u) \lesseqgtr \frac{2\xi n_w(1-u) - n_e A_\theta}{2\xi} \Leftrightarrow (A_\theta n_e - \xi n_w(1-u))^2 + 4\xi n_e \gamma \lesseqgtr (n_e A_\theta - (A_\theta n_e - \xi n_w(1-u)))^2.$$

Further manipulation reveals that

$$g_\theta^1(u) \lesseqgtr \frac{2\xi n_w(1-u) - n_e A_\theta}{2\xi} \Leftrightarrow 4\xi\gamma \lesseqgtr A_\theta \left(2\xi n_w(1-u) - n_e A_\theta\right).$$

Using the fact that $n_w = 1 - n_e$, we conclude that

$$\tau_\theta^1(u) \lesseqgtr \tau_\theta^* \Leftrightarrow n_e \lesseqgtr \frac{1 - u - 2\gamma/A_\theta}{1 - u + A_\theta/2\xi}.$$

■

Table 1

Empirical relation between public debt/GDP and unemployment

-OECD countries, 2006-2010-

VARIABLES	(1) unem	(2) unem	(3) unem	(4) absunem	(5) absunem	(6) absunem
debt	0.0762***	0.0751***	0.0787***	0.0269***	0.0243***	0.0258***
	(0.0179)	(0.0182)	(0.0161)	(0.00799)	(0.00825)	(0.00784)
debt_plus_90			-0.00712			-0.00565
			(0.00884)			(0.00497)
l_dependency	0.0804	0.0819	0.0925	0.217	0.208	0.218
	(0.358)	(0.356)	(0.364)	(0.245)	(0.232)	(0.230)
l_popgrowth	-1.612*	-1.566*	-1.668*	0.842*	0.775*	0.784*
	(0.900)	(0.883)	(0.958)	(0.494)	(0.429)	(0.428)
l_open	-0.0315	-0.0326	-0.0293	-0.0299	-0.0317	-0.0306
	(0.0293)	(0.0289)	(0.0263)	(0.0208)	(0.0230)	(0.0228)
l_bond		0.287	0.287		0.471**	0.508**
		(0.279)	(0.293)		(0.214)	(0.207)
Constant	1.811	1.775	0.929	-9.394	-10.35	-11.51
	(16.77)	(16.17)	(16.60)	(11.88)	(11.31)	(11.22)
Observations	150	150	150	150	149	149
R-squared	0.611	0.615	0.618	0.387	0.410	0.414
Number of countries	30	30	30	30	30	30

Notes: OLS estimation results. Columns 1-3: u(t) = a + b*debt(t) + c*controls(t-1) + error. Columns 4-6: |(u(t)-u(t-1)|= a + b*debt(t-1) + c*controls(t-2) + error. Country and year fixed effects are included. Standard errors (in parentheses) are clustered by country. *** p<0.01, ** p<0.05, * p<0.1.