Property Taxation, Zoning, and Efficiency in a Dynamic Tiebout Model*

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Abstract

This paper presents a new dynamic Tiebout model and uses it to revisit a classic argument in state and local public finance. The argument, due to Hamilton (1975), is that a system of local governments financing service provision via property taxes will produce an efficient allocation of both housing and services if governments can implement zoning ordinances. In the model, it is shown analytically that when local governments choose zoning along with taxes and services there does not exist an equilibrium that is both efficient and satisfies a local stability property. It also shown using numerical methods that there exists an equilibrium in which governments over-zone and households are forced to over-consume housing. These findings challenge the Benefit View of the property tax.

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1 Introduction

This paper presents a new dynamic Tiebout model and uses it to revisit a classic issue in state and local public finance. The importance of introducing dynamics into Tiebout models has long been recognized, but the problem is challenging.[1] The model presented here, in standard Tiebout fashion, features multiple communities and households who choose between them. The size of each community is determined by its housing stock which can vary over time. Housing stocks are variable because new construction is possible and, in each period, part of a community's housing stock is lost to depreciation. Housing is owned by residents and new construction is supplied by a competitive construction sector. Public policies in each community are chosen each period by the residents of the community. When choosing policies, residents anticipate how they will impact their property values as well as the future composition of their communities.

The model has the potential to be used to analyze numerous issues in state and local public finance. The issue focused on in this paper is Hamilton's argument that a system of local governments financing public service provision via property taxes will produce an efficient allocation of both housing and services if local governments can implement zoning ordinances (Hamilton 1975).[2] This is not only an interesting argument from a theoretical viewpoint but it has also been very influential. In particular, it has given rise to the so-called Benefit View which argues that the property tax should be seen as a non-distortionary and non-redistributive user charge for public services.[3] The results from our model highlight forces that work against Hamilton's argument and provide a novel critique of the Benefit View.

To explain what the model allows us to do and the new forces it reveals, it is helpful to briefly review Hamilton's argument. Note first that his conclusion is surprising since basic economic intuition suggests property taxes will distort both housing choices and public

[1] There are only a small number of papers considering dynamic models in which households choose between communities. Examples include Benabou (1996), Conley, Driskill, and Wang (2013), Epple, Romano, and Sieg (2012), and Glomm and Lagunoff (1999). Moreover, in these models, either there is no housing (i.e., households occupy a piece of land) or the supply of housing in the communities is exogenous.

[2] In the U.S., public services like education, police, and libraries, are provided by local governments whose primary source of revenue has traditionally been the property tax. Given this, the efficiency properties of a system of local governments financing public service provision via property taxes have long been of interest. For a history of the use of the property tax in the U.S. see Wallis (2001).

[3] The Benefit View is distinct from the *New View* that argues that the property tax encourages capital to move away from the housing sector to other sectors. This shift produces a lower quality housing stock in the long run. Moreover, if the national supply function of capital is upward sloping, the equilibrium price of capital will fall so that some of the incidence of the property tax will fall on capital owners. For a formal exposition of the New View see Zodrow and Mieszkowski (1986). For discussion and further debate see Fischel (1992, 2001a, 2001c), Mieszkowski and Zodrow (1989), Nechyba (2001) and Zodrow (2001a, 2001b).

service levels. Housing choices will be distorted because households will seek lower quality houses to avoid taxation. Public service levels will be distorted because households will base their demand for services on the tax price which will not typically reflect the true price. If the decisive voter in a community has a house less valuable than the average property, his tax price will be lower than the true price, and services will be over-provided. By contrast, services will be under-provided when the decisive voter's house is of above average value.

Hamilton's argument is that through zoning, local governments can establish minimum housing qualities for their communities. Households then sort into communities based on their desired housing quality levels. Nobody will build a higher quality house than the minimum permitted, since they would be better off building such a house in a more tightly zoned community in order to get a lower tax price. Thus, in equilibrium, communities will comprise of homogeneous properties. Homogeneity implies that in equilibrium, in all communities, the tax price faced by voters equals the true price and services will be provided efficiently. Property taxes will be benefit taxes in that each household's tax bill exactly equals the cost of the services it consumes. Benefit taxes lead households to choose between communities efficiently.

Hamilton does not specify a precise model of how communities set their zoning ordinances and public service levels. Rather, he simply assumes that households face a set of communities offering a full range of policies. As first noted by White (1975), this begs the question of what options would be available to households in equilibrium. While subsequent literature has attempted to clarify the issue, the task is difficult because the problem has a natural dynamic structure. Zoning ordinances are chosen by existing residents and impact only new construction. It is therefore through its effect on new construction that zoning determines the composition of communities and housing prices. However, the literature employs static models in which distinctions between existing and future residents and old and new construction are hard to capture.

The dynamic structure of the model presented in this paper allows the impact of zoning on both new construction and the existing stock of housing to be captured. In particular, it captures the key grandfathering characteristic of zoning whereby existing property is exempt from regulation. When choosing zoning, existing residents anticipate how it will impact the value of their properties and also the tax price and level of services in their community. Existing residents would like to boost the value of their homes, while at the same time lowering their tax price of services and keeping service levels in line with their preferences. The paper shows that the incentives created by these considerations imply that there does not exist an equilibrium with endogenous zoning which has a steady state that is efficient and satisfies a local stability property. This result directly contradicts Hamilton's argument and the Benefit View of the property tax.

The basic intuition for this result is simple. In an efficient steady state communities will be stratified according to their housing qualities. A full mix of housing qualities is necessary for efficiency. Consider the community with the lowest quality housing and imagine that it deviated from the equilibrium by imposing more stringent zoning. In the short run, this would raise the prices of existing low quality homes by restricting supply. It would have no adverse effect on the tax price of services in the community, because the only homes that could be built in the community would be of higher quality than the existing stock and thus command a higher price. Thus, in the short run all existing residents would benefit from deviating from the equilibrium. In the long run, matters are more complicated because the deviation could create future policy changes in other communities that might be harmful. However, if the equilibrium satisfies a local stability property, such harmful future policy effects can be ruled out.

This negative finding naturally raises the question of what will happen in the long run when zoning decisions are endogenous. To shed light on this question, the paper develops a numerical approach to compute equilibria of the model. With the parameters set at empirically reasonable values, an equilibrium exists which exhibits over-zoning. In this equilibrium, households are forced to over-consume housing in the long run. Equilibrium welfare is actually lower with zoning than without. Interestingly, in this equilibrium, communities are homogeneous in steady state so that property taxes are benefit taxes and service levels are efficient. This component of the Benefit View of the property tax is therefore upheld. The problem is that housing decisions are distorted directly by zoning.

As noted earlier, the model presented here could be applied to address many different issues. We focus on zoning because it is a relatively simple policy with obvious dynamic consequences. There are many other policies that communities have to choose with dynamic consequences. These include investments in durable public goods (parks, roads, schools, etc), bond issues, subsidies/taxes on new construction, and subsidies to attract firms to locate in the community. When considering such policies, residents will have a keen eye on how they impact their property values as well as their tax bills and benefits from public goods and services. This is precisely what the model can capture.

The organization of the remainder of the paper is as follows. Section 2 identifies related literature and Section 3 introduces the model. Sections 4 and 5 set the stage by discussing equilibrium with no zoning and with exogenous zoning. The heart of the paper is Section 6 which considers endogenous zoning. This section presents the inefficiency result, the numerical analysis, and discusses the implications of the findings for the Benefit View. Section 7 argues that the results are robust to allowing more communities. Section 8 identifies some of the other issues the model could be used to analyze and Section 9 concludes.

2 Related literature

The paper relates to two distinct literatures. The first is the state and local public finance literature on Tiebout models. In a seminal paper, Tiebout (1956) suggested that the mechanism of households "voting with their feet" by choosing between communities on the basis of their public service-tax packages could improve allocative efficiency. His idea was that households would sort into communities with others who had similar demands for services and this sorting would create gains in public service surplus. Since then, a large theoretical literature has developed exploring this basic idea.[4]

Tiebout's analysis assumed that local governments financed service provision by head taxes. Since head taxes are rarely part of the public finance landscape, the literature quickly developed Tiebout models incorporating property tax finance. There are two varieties, distinguished by their assumptions about housing supply. Both assume that households have preferences defined over housing, private consumption, and public services, that communities finance service provision by a proportional tax on housing, and that service levels are chosen collectively by residents. The first variety assumes that housing is supplied by absentee landlords according to exogenously given supply schedules (for example, Epple, Filimon and Romer 1984). In these models, households are best interpreted as renters. The second variety assumes that the housing stock is fixed and owned by residents (for example, Hamilton 1976 and Nechyba 1997). Property taxation complicates Tiebout sorting because the tax price of services is below the true price for households who consume relatively less housing. This makes it attractive for households to live in communities where they consume relatively less housing. In the first variety, this force makes it difficult to find stable allocations of households across communities. In the second, it results in capitalization: small houses in communities with a larger fraction of large houses cost more.[5]

The dynamic model presented here builds on these static Tiebout models with property taxation. It follows the second variety in assuming existing houses are owned by residents. However, new houses can be built by competitive construction firms. In the spirit of the first variety, the location of new construction is influenced by the existing mix of homes in the

[4] For an excellent review of this literature see Ross and Yinger (1999).

[5] The difficulties in finding stable allocations in the first variety of model can be overcome with appropriate assumptions. Epple, Filimon and Romer (1993) prove the existence of equilibrium in a model in which households have identical preferences and different income levels. Equilibrium involves communities stratified by income levels. Lower income households do not wish to live in higher income communities even though the tax price of services is lower, because the overall spending on services is higher than they would like. As shown by Nechyba (1997), in the second variety of model equilibrium exists under general conditions.

communities as this determines the tax price of services.

The second related literature is that on zoning.[6] The traditional justification for zoning is to deal with externalities. Such externality zoning can, for example, prevent over-crowding of communities. However, it has long been recognized that zoning can be employed to alter the allocation of the costs of public services between existing residents and newcomers or outsiders, a practice known as fiscal zoning.[7] Hamilton's work highlights a normatively attractive aspect of such zoning by showing how it could be used to overcome the problem of newcomers paying less than their fair share of services by buying cheaper houses. But, as emphasized by White (1975), zoning might also be used to force newcomers to pay more than their fair share, a practice she refers to as fiscal-squeeze zoning. In addition, zoning could be used by existing residents to increase the value of their homes by restricting supply, which White calls scarcity zoning. These abuses of zoning might give rise to over-zoning (Davis 1963), whereby the supply of housing is inefficiently restricted. This paper's dynamic model captures the distributional conflict between existing residents and newcomers, and permits a unified treatment of the use of zoning to manipulate both housing prices and the surplus obtained from public services.

There are two prior studies of zoning in Tiebout models with property taxes and these represent the closest antecedents to this paper.[8] Fernandez and Rogerson (1997) study the impact of zoning in a two-community model of the variable housing supply variety. They assume households differ in their income levels and that housing and public services are normal goods. They model zoning as a minimum housing level and assume that only one community imposes it. They first study the impact of an exogenous zoning requirement and then analyze an endogenously determined level. They assume residents first choose a community to live in, then collectively choose policies, and finally choose housing and private consumption. They study how the incorporation of zoning impacts allocations and household welfare.

Fernandez and Rogerson choose parameters so that, without zoning, their two communities are stratified by income levels. Their numerical analysis suggests that the introduction of exogenous zoning that makes the richer community more exclusive benefits the richest households and hurts the poorest. For households in the middle of the income distribution, welfare changes are complex and non-monotonic in income. With endogenous zoning, they

[6] For an excellent introduction to zoning and other land-use regulations see Fischel (1999).

[7] See, for example, Margolis (1956). Zoning may also be motivated by the desire to change the type of household entering the community. Residents may believe requiring new houses to have large lots will attract a better class of resident. This may reduce crime and yield better peer groups in schools. This is referred to as *exclusionary zoning* and is analyzed in Oates (1977) and Calabrese, Epple and Romano (2006).

[8] See also Pogodzinski and Sass (1994) who present a theoretical framework to underpin their empirical investigation of the impact of zoning on housing values.

discover a potential problem with the existence of a majority-preferred zoning level in the richer community. When equilibrium exists, their numerical analysis suggests that the richer community chooses a zoning level that makes it more exclusive than without zoning. Taxes and the quality of public services rise in both communities, as zoning shifts the lower income households from the richer community into the poorer community. The welfare effects are as in the case of exogenous zoning.

A limitation of the Fernandez and Rogerson analysis, is that communities are fixed by the time zoning decisions are made so that their impact on community composition is not captured. To address this, Calabrese, Epple and Romano (2007) incorporate zoning in a different way. Households first choose an initial community of residence, which is committed by a purchase of land and then residents collectively choose zoning and property taxes for their communities. Existence problems are dealt with by assuming that residents elect a leader to choose policy rather than directly vote over policies. After these policy choices, households revisit their choice of community and purchase housing and consume public services in their new community. This quasi-dynamic structure means that residents anticipate the impact of their decisions on land prices and community composition.

Calabrese, Epple and Romano also assume that households differ in their income levels and that housing and public services are normal goods. However, they choose parameters so that their communities are identical without zoning. Stratification by income levels does not take place because poor households prefer to live in rich communities to benefit from the lower tax price of services. With zoning, there is stratification, with higher income communities having stricter minimum housing requirements. Calabrese et al's numerical analysis shows that zoning leads to aggregate welfare gains because it reduces distortions not only in public service provision but also in housing consumption. The richest households benefit from zoning, while the poorest are made worse off. In the spirit of Hamilton (1975), the equilibrium with zoning generates an aggregate welfare level that is only slightly lower than that arising when communities can impose head taxes.

The advance of this paper's model over these works lies in its dynamic structure. This allows the impact of zoning on the value of the existing stock of housing to be captured. While in Calabrese et al's quasi-dynamic set-up households anticipate how zoning impacts the value of their land, all housing is produced after zoning ordinances have been decided. The dynamic structure also allows the key grandfathering characteristic of zoning whereby existing property is exempt from regulation to be captured. By contrast, in the models of Fernandez and Rogerson and Calabrese et al, households are bound by the constraints that they impose.

3 A dynamic Tiebout model

3.1 The model

Consider a geographic area consisting of two communities, indexed by $i \in \{1, 2\}$. The time horizon is infinite, with periods indexed by $t \in \{0, ..., \infty\}$. A constant population of households of size 1 need to reside in the area, but there is turnover, so that in each period new households arrive and old ones leave. The probability that a household residing in the area will need to remain there in the subsequent period is μ. Thus, in each period, a fraction $1 - \mu$ of households leave the area and are replaced by an equal number of new ones.[9]

The only way to live in the area is to own a house in one of the communities.[10] Houses come in two types, large and small. Houses are durable, but a fixed fraction d of the stock in each community is destroyed at the beginning of each period.[11] This fraction is assumed to be less than $1 - \mu$, so that households face a higher probability of having to leave the area than of having their houses destroyed. New houses can be built in each period and the cost of building a house of type $H \in \{L, S\}$ is C_H where C_L exceeds C_S. Each community has enough land to accommodate a population of size 1 and land has no alternative use.[12] The stock of houses of type H in community i at the beginning of a period is denoted O_{Hi} and new construction is denoted N_{Hi}. Construction takes place at the beginning of each period following the destruction of existing homes. Thus, post construction, there are $(1-d)O_{Hi}+N_{Hi}$ type H houses in community i. New and old houses are perfect substitutes.

A public service is provided in each community. The service level in community i is denoted g_i. The cost of the service is cg_i per household.

Each household receives an exogenous income of y per period.[13] When living in the area, households have preferences defined over housing, public services, and private consumption. They differ in their preferences for large houses which are measured by the parameter θ.

[9] We have in mind that households leave for reasons to do with employment opportunities or changes in family circumstance.

[10] The model does not "micro-found" why households cannot rent houses. The usual assumption is that moral hazard issues in the maintenance of the house make owning the more efficient arrangement. Obviously, if households were renters they would have different incentives with respect to property values. On these issues see Ortalo-Magne and Prat (2011).

[11] This assumption follows Glaeser and Gyourko (2005) and is necessary to get turnover of the housing stock. Given the constant population, if all houses were infinitely durable there would be no dynamics. By housing being "destroyed", we have in mind both literal destruction by floods, hurricanes, fires, or termites, and also houses being torn down because of decay due to the passage of time.

[12] This implies that the supply of housing in each community is perfectly elastic over the relevant range which is in the spirit of Hamilton's assumptions. The model can be extended to allow land not used for housing to have some constant productivity in agricultural use. This complicates notation without fundamentally changing the insights from the analysis.

[13] There are no income effects, so income heterogeneity can be introduced without changing the results.

A household of type θ with private consumption x and services g obtains a period payoff of $\theta + x + B(g)$ if it lives in a large house and $x + B(g)$ if it lives in a small house. The service benefit function $B(g)$ is twice continuously differentiable, increasing, strictly concave, and satisfies the Inada conditions. When not living in the area, a household's payoff just depends on its private consumption. Households discount future payoffs at rate δ and can borrow and save at rate $1/\delta - 1$. The range of preference types is $[0, \overline{\theta}]$ and the distribution is described by the cumulative distribution function $F(\theta)$. This function is assumed to be increasing and continuously differentiable on the interval $[0, \overline{\theta}]$ and the upper bound of the support $\overline{\theta}$ is assumed to exceed $(1 - \delta(1-d))(C_L - C_S)$. These assumptions imply positive demand for both types of houses in an efficient allocation.

There are competitive housing markets in both communities which open at the beginning of each period. Demand comes from new households moving into the area and remaining residents who need new houses or who want to move. Supply comes from owners leaving the area, residents who want to move, and new construction. Construction is supplied by competitive construction firms. The price of houses of type H in community i is denoted P_{Hi}. The price P_{Hi} can fall below the replacement cost C_H if demand at this price falls short of the existing stock $(1-d)O_{Hi}$.

Service provision in each community i is financed by a proportional tax τ_i on the value of property $\sum_H P_{Hi}\left[(1-d)O_{Hi} + N_{Hi}\right]$. Each community must balance its budget in each period implying that

$$\tau_i \sum_H P_{Hi}\left[(1-d)O_{Hi} + N_{Hi}\right] = cg_i \sum_H \left[(1-d)O_{Hi} + N_{Hi}\right] \qquad i \in \{1,2\}. \tag{1}$$

The level of service provision in any period is chosen collectively by the residents of the community that period. The service level preferred by a majority of residents is implemented.

The timing of the model is as follows. Each period begins with a stock of houses $O = (O_{L1}, O_{S1}, O_{L2}, O_{S2})$ of aggregate size 1. At the beginning of the period, a fraction d of the housing stock is destroyed.[14] In addition, existing residents learn whether they will be remaining in the area and new households join the pool of residents.[15] Housing markets open, housing prices $P = (P_{L1}, P_{S1}, P_{L2}, P_{S2})$ are determined, and new construction $N = (N_{L1}, N_{S1}, N_{L2}, N_{S2})$ takes place. The total amount of construction must equal d.[16] The

[14] Each house is equally likely to be destroyed. While it would be more realistic to assume that older houses were more likely to be destroyed, introducing this feature would require keeping track of the age of each house and allowing for age-dependent housing prices. This would make the model much less tractable.

[15] It should be stressed that the new households have the same θ-type distribution as those leaving the pool.

[16] Underlying this assertion is the implicit assumption that the demand for large houses is not sufficiently strong that a significant fraction of buyers would prefer to buy a large house at price C_L than take a small

housing market activity determines the post-construction housing stocks $(1-d)O+N$. Residents then choose the public service levels g_1 and g_2 which determine the property tax rates τ_1 and τ_2. The next period begins with the stock of houses $O' = (1-d)O+N$.

3.2 Discussion

The model just presented makes many simplifying assumptions. Some of these are made to craft our critique of Hamilton's argument in the most economical manner and can easily be relaxed. Others are more fundamental to the approach. In the former category is the assumption that households have identical preferences for public services. This assumption is made because it allows us to address Hamilton's argument in a two community model. Recall that Hamilton assumes that there are enough communities to accommodate each possible desired housing-public service bundle. With two house types and uniform public service preferences, two communities are sufficient. With two types of public service preferences, we need four communities; with three types, we need six; etc. Of course, for many questions in state and local public finance, it is essential to allow households to have different preferences for the public service. This can be done by introducing an additional taste parameter ζ and assuming that a type (θ, ζ) household with private consumption x and services g obtains a period payoff of $\theta + x + \zeta B(g)$ if it lives in a large house and $x + \zeta B(g)$ if it lives in a small house.

A further assumption made just to be consistent with Hamilton is that the cost of providing services to a household is independent of the size of the community. This assumption eliminates concerns about optimal community size. It is important for Hamilton's argument because it allows households with different desired housing-public service bundles to be accommodated in their own communities. If say, the average cost of providing services were u-shaped, different household types may need to be combined together to realize economies of scale in service provision. This would directly undermine Hamilton's argument. It is certainly possible to introduce u-shaped average costs and other agglomeration effects into the model and we discuss this further in Section 8.

A more fundamental assumption is that utility is linear in private consumption. This assumption implies that households do not care about risk or the inter-temporal allocation of their consumption. As we will see, it makes households' demand for housing very simple in that it just depends on this and next period's house prices, this period's taxes and public services, and their tastes for housing. This simplicity permits a clean focus on the collective decisions of communities. Without this assumption, a household's demand for housing and

house for a zero price.

public services would depend upon its wealth which would in turn depend on the value of its house. Accounting for each household's wealth in the set of state variables would make the model much less tractable.

One consequence of this linearity assumption is that it removes income effects from the model. This jars with the literature since the sorting of different income groups across communities is a major focus of static Tiebout models with property taxation. Indeed, the standard assumption in the literature is that households differ only in their income levels and that housing and public services are normal goods. Under appropriate conditions, this assumption will yield communities stratified by income levels. Nonetheless, similar results can be obtained in this model by introducing heterogeneity in both housing and public service preferences as discussed above and interpreting households with stronger preferences for housing and public services as higher income households.

4 Equilibrium with no zoning

We begin our study of the model by analyzing what would happen without zoning. This will clarify the distortions created by property taxation that zoning is supposed to overcome. We first define what is meant by an equilibrium and then discuss some properties of equilibrium. Next we study steady states and discuss the existence of equilibrium and convergence to these steady states. Finally, we discuss the efficiency of these steady states.

4.1 Definition of equilibrium

The model has a recursive structure. The state can be summarized by the stock of houses O.[17] Given this stock, the housing market determines prices and new construction and we recognize this dependence by writing $P(O)$ and $N(O)$. The prices $P(O)$ and post-construction housing stock $(1-d)O+N(O)$ then determine the tax bases of the two communities and these in turn determine public service levels $(g_1(O), g_2(O))$ and tax rates $(\tau_1(O), \tau_2(O))$. Households understand what prices, construction, services, and taxes will be given any initial state O. They also understand that next period's stock of houses will be given by $O' = (1-d)O+N(O)$. They treat all these aggregate relationships as exogenous and beyond their control.

[17] Under our assumptions of no income effects and costless mobility, the allocation of homes among households does not impact market outcomes or policy determination. Thus, it is not necessary to keep track of this allocation as a state variable.

Decisions of households At the beginning of any period, after housing has been destroyed and the pool of residents determined, households fall into two groups: those who resided in the area in the previous period and those who did not, but must in the current period. The first group is differentiated by the homes they own. There are five possible home ownership states represented by $o \in \{L1, S1, L2, S2, \varnothing\}$; $o = Hi$ means that the household owns a type H house in community i and $o = \varnothing$ means that it does not own a house (which would be the case if its house was destroyed). The second group of households will not own homes, so that $o = \varnothing$ for all these households.

Households in the first group who need to leave the area will sell their houses and obtain a continuation payoff of

$$P_o(O) + \frac{y}{1-\delta}, \tag{2}$$

where $P_{\varnothing}(O) = 0$. The remaining households in the first group and all those in the second must decide in which community to live and in what type of house. Formally, they must make a home ownership decision $a \in \{L1, S1, L2, S2\}$. Since selling a house and moving is costless and houses of the same type are perfect substitutes, there is no loss of generality in assuming that all households owning houses at the beginning of any period sell them.[18] By this logic, each household's home ownership decision is independent of its home ownership state o. Moreover, the only future consequences of the current period choice of housing is through the selling price in the subsequent period.

To make this more precise, let $V_\theta(O)$ denote the expected payoff of a household of type θ at the beginning of a period in which it has to live in the area, does not own a house, and the aggregate state is O. Then, the expected payoff of a household of type θ at the beginning of a period in which it has to live in the area and is in home ownership state o is

$$P_o(O) + V_\theta(O). \tag{3}$$

The value function $V_\theta(O)$ satisfies the functional equation

$$V_\theta(O) = \max_{a \in \{L1,S1,L2,S2\}} \left\{ \begin{array}{c} y + \theta I_{H(a)} + B(g_{i(a)}(O)) - \tau_{i(a)}(O)P_a(O) - P_a(O) \\ +\delta[(1-d)P_a(O') + \mu V_\theta(O') + (1-\mu)\frac{y}{1-\delta}] \end{array} \right\}, \tag{4}$$

where I_H is an indicator function equal to 1 if H equals L, $H(a)$ is the house type associated

[18] It should be stressed that this is just a convenient way of understanding the household decision problem. The equilibrium we study is perfectly consistent with the assumption that the only households selling their homes are those who plan to leave the community or who desire a different size house. Households who keep their houses can be thought of as selling their house each period and buying it right back and for most households this will be consistent with optimal decision-making. In equilibrium, the fraction of households who wish to change the size of their house is bounded by d.

with home ownership choice a, and $i(a)$ is the community associated with a.[19] Let $\alpha_\theta(O)$ be the set of optimal home ownership choices. This will contain more than one element if, for example, households are indifferent between communities.

Housing market equilibrium Construction firms are competitive and the production costs of new homes are constant. Accordingly, the supplies of large and small homes are perfectly elastic at the prices C_L and C_S, respectively. This means that if $P_{Hi}(O)$ equals C_H, firms will willingly supply any number of new homes of type H in community i but if $P_{Hi}(O)$ is less than C_H none will be supplied.

Let $\xi_{Hi}(\theta, O)$ be the fraction of type θ households selecting houses of type H in community i and let $\xi(\theta, O)$ denote the vector $(\xi_{L1}(\cdot), \xi_{S1}(\cdot), \xi_{L2}(\cdot), \xi_{S2}(\cdot))$. If a positive fraction of type θ households are selecting houses of type H in community i it must be the case that Hi is in the set of optimal choices for these households; i.e., $Hi \in \alpha_\theta(O)$. In equilibrium, it must be the case that the total fraction of households selecting houses of type H in community i is equal to the supply of such houses; that is,

$$\int_\theta \xi_{Hi}(\theta, O) dF(\theta) = (1-d)O_{Hi} + N_{Hi}(O). \tag{5}$$

In addition, it must be the case that all households of type θ are selecting some type of housing, so that for all types θ we have that

$$\sum_H \sum_i \xi_{Hi}(\theta, O) = 1. \tag{6}$$

Choice of public service levels and tax rates All households get the same benefit from public services. However, residents living in different houses face different tax prices for services, which may give rise to different preferred service levels. Using (1), the preferred service level for residents of type H houses in community i is

$$g^*(\rho_{Hi}(O)) = \arg\max_g \{B(g) - \rho_{Hi}(O)g\}, \tag{7}$$

[19] This formulation embodies the implicit assumption that the exogenous income stream of the household is sufficient to finance his desired housing purchase. One objection to this assumption concerns what would happen to a household who bought a sequence of houses all of which were destroyed. This objection can be dealt with by noting that nothing in the model would change if we assumed that all households purchased actuarially fair insurance against the loss of their houses. This reflects the fact that households are risk neutral.

where $\rho_{Hi}(O)$ is the tax price of services that residents of type H houses in community i face when the state is O. This tax price is given by

$$\rho_{Hi}(O) = \frac{cP_{Hi}(O)}{P_{Li}(O)\lambda_i(O) + P_{Si}(O)(1-\lambda_i(O))}, \tag{8}$$

where $\lambda_i(O)$ is the fraction of post-construction houses that are large in community i. The tax price is determined by the relative price of type H houses in community i and the fraction of large houses. If large houses are more expensive than small houses, the tax price is lower for those owning small houses and is decreasing in the fraction of large houses. The majority preferred level of public services in community i is

$$g_i(O) = \begin{cases} g^*(\rho_{Li}(O)) & \text{if } \lambda_i(O) \geq 1/2 \\ g^*(\rho_{Si}(O)) & \text{if } \lambda_i(O) < 1/2 \end{cases}. \tag{9}$$

Using (1), the associated tax rate is

$$\tau_i(O) = \frac{cg_i(O)}{P_{Li}(O)\lambda_i(O) + P_{Si}(O)(1-\lambda_i(O))}. \tag{10}$$

The simplicity of these optimal policies reflects the assumption that policies are chosen after the market for housing has cleared. At that point, the housing stock and its value are predetermined. When chosen, therefore, the property tax is a non-distortionary tax on capital and equilibrium responses are irrelevant for the calculus of citizen decision-making. While taxes and public services do impact the housing market, it is the expectation of these taxes and services that are relevant and the taxes chosen this period do not influence expectations concerning next period's taxes given the Markovian structure of the equilibrium. This contrasts with Tiebout models with property taxation of the variable housing supply variety which assume taxes are chosen before housing choices are made.[20]

Equilibrium An equilibrium with no zoning consists of a price rule $P(O)$, a new construction rule $N(O)$, public service rules $(g_1(O), g_2(O))$, tax rules $(\tau_1(O), \tau_2(O))$, and, for each household type, a value function $V_\theta(O)$, a housing demand correspondence $\alpha_\theta(O)$ and housing selection functions $\xi(\theta, O)$, such that three conditions are satisfied. First, household optimization: for each household type θ the value function $V_\theta(O)$ satisfies (4) and, for all

[20] In this spirit, an alternative modelling assumption would be that in each period contemporaneous property taxes are fixed and households vote on next period's taxes. Households would then anticipate how next period's taxes would impact next period's housing market equilibrium. While this assumption is perhaps less natural than the assumption made here (and certainly more complicated), it would be interesting to work out its implications.

O, every element of $\alpha_\theta(O)$ solves the maximization problem described in (4). Second, housing market equilibrium: the housing selection functions and new construction rules satisfy (5) and (6), and, in addition, $\xi_{Hi}(\theta, O)$ is positive only if Hi is an element of $\alpha_\theta(O)$ and $N_{Hi}(O)$ is positive only if $P_{Hi}(O) = C_H$. Third, majority rule: the public service and tax rules satisfy (9) and (10).

4.2 Some properties of equilibrium

Inspecting the household's problem (4) and using the definitions in (8) and (10), it is clear that a household choosing a type H house will prefer to live in the community that maximizes[21]

$$[B(g_i) - \rho_{Hi} g_i] - P_{Hi} + \delta(1-d)P'_{Hi}. \tag{11}$$

The term in square brackets is the public service surplus associated with community i, which is the difference between service benefits and the tax cost. The second term is the current price of a type H house in community i and the final term is the discounted expected value of the house next period. Notice that (11) is independent of θ, so that all households choosing type H houses have the same preferences over communities. Thus, in equilibrium, if type H houses are available in both communities, all those choosing them must be indifferent between communities. It follows from (11) that for $H \in \{L, S\}$

$$[B(g_1) - \rho_{H1} g_1] - P_{H1} + \delta(1-d)P'_{H1} = [B(g_2) - \rho_{H2} g_2] - P_{H2} + \delta(1-d)P'_{H2}. \tag{12}$$

This arbitrage equation implies that differences in public service surplus across communities must be capitalized into differences in housing prices as argued by Hamilton (1976).

From the household's problem (4), we also see that a household will prefer a large house in community i to a small house if its preference θ exceeds

$$P_{Li} - P_{Si} - \delta(1-d)(P'_{Li} - P'_{Si}) + (\rho_{Li} - \rho_{Si})g_i. \tag{13}$$

This expression represents the higher cost of a large house and includes both price and tax differences. Note that (12) implies that (13) is equalized across communities. Thus, letting

$$\theta_c = P_{Li} - P_{Si} - \delta(1-d)(P'_{Li} - P'_{Si}) + (\rho_{Li} - \rho_{Si})g_i, \tag{14}$$

it follows from (12) and (13) that all households with preference larger than θ_c will prefer a

[21] The term P'_{Hi} is short-hand for $P_{Hi}(O')$. Similarly, P_{Hi} is short-hand for $P_{Hi}(O)$, g_i is short-hand for $g_i(O)$, etc.

large house and all those with preference less than θ_c a small house. In equilibrium, therefore, we must have that

$$F(\theta_c) = \sum_{i=1}^{2}((1-d)O_{Si} + N_{Si}), \tag{15}$$

and that

$$1 - F(\theta_c) = \sum_{i=1}^{2}((1-d)O_{Li} + N_{Li}). \tag{16}$$

Exactly how types with preference larger than θ_c are allocated across the two communities does not matter, provided that (5) and (6) are satisfied. Similarly, for types with preference smaller than θ_c.

4.3 Steady states

Given an equilibrium, a stock of houses O^* is a steady state if new construction at O^* is such as to maintain the stock constant. Our first proposition tells us what steady states look like. We begin by imposing an assumption on the demand for public services. To state this, let $\varepsilon_d(\rho)$ denote the elasticity of demand for public services at tax price ρ.[22] Then, we make:

Assumption 1 *For all* $\rho \in [c, \frac{2C_L}{C_L+C_S}c]$

$$\varepsilon_d(\rho) < \frac{1}{1-\frac{C_S}{C_L}}.$$

This assumption bounds the elasticity of demand for public services over a range of tax prices. The literature on the demand for public services stemming from the work of Bergstrom and Goodman (1973) suggests that service demand is inelastic and thus this is an innocuous assumption.

Proposition 1 *Suppose that Assumption 1 is satisfied and let O^* be a steady state of an equilibrium with no zoning. Then, the fraction of large houses in each community is the same; that is, $\lambda_1(O^*) = \lambda_2(O^*) = \lambda^*$. If this fraction exceeds $1/2$, the public service level in each community is $g_L^* \equiv g^*(\frac{cC_L}{C_L\lambda^*+C_S(1-\lambda^*)})$ and households live in large houses if their preference exceeds*

$$(1-\delta(1-d))(C_L - C_S) + \frac{c(C_L - C_S)}{C_L\lambda^* + C_S(1-\lambda^*)}g_L^*. \tag{17}$$

If the fraction is less than $1/2$, the service level is $g_S^* \equiv g^*(\frac{cC_S}{C_L\lambda^*+C_S(1-\lambda^*)})$ and households

[22] That is, $\varepsilon_d(\rho) = -\frac{\rho}{g^*(\rho)}\frac{dg^*(\rho)}{d\rho}$.

live in large houses if their preference exceeds

$$(1-\delta(1-d))(C_L - C_S) + \frac{c(C_L - C_S)}{C_L\lambda^* + C_S(1-\lambda^*)} g_S^*. \tag{18}$$

To understand the proposition, suppose first that in steady state both communities have both types of houses. Then, since there must be new construction of both types of houses in both communities, steady state housing prices equal construction costs. This implies that the fraction of large houses in each community must be the same. For if one community had a greater fraction of large houses, the public service surplus enjoyed by large house owners in that community would be higher than in the other, violating arbitrage condition (12). Since both house prices and the fraction of large houses are the same across the communities, it follows that service levels and taxes are also the same. From (7), if a majority of households own large houses, the public service level will be g_L^* and, from (14), households live in large houses only if their preference exceeds the expression in (17). The first term in this expression reflects the additional resource cost of a large house in per-period terms, while the second reflects the extra tax cost of services for large home owners.[23] If a majority of households own small houses, the public service level is g_S^* and households live in large houses only if their preference exceeds the expression in (18).

This argument presumes that in steady state both communities have both types of houses. This ignores the possibility that one community has a mix of large and small houses and the other has only small houses.[24] At first glance, such a situation seems inconsistent with equilibrium because small home owners would enjoy a lower tax price in the mixed community. This ignores the fact that if the mixed community has a majority of large houses, the public service level will be chosen by large home owners. Households in the small house community might then be compensated for the higher tax price by getting their preferred public service level. In this way, an asymmetric steady state is in principle possible. However, the conditions under which it exists are demanding. In particular, it cannot exist under Assumption 1.[25] Intuitively, it is only when the demand for services is highly elastic that the gain in surplus for a household in the small house community created by getting its preferred service level can compensate for the higher tax price.

[23]The additional resource cost of a large house is $C_L - C_S$. However, the house will be available for use in the next period with probability $1-d$ and this will save society spending $C_L - C_S$ then. The present expected value of these saved resources is $\delta(1-d)(C_L - C_S)$. The per-period additional resource cost of a large house is therefore $(1-\delta(1-d))(C_L - C_S)$.

[24]There are also other possibilities (for example, one community has only large houses and the other has a mix), but these are more easily ruled out. The formal proof of Proposition 1 provides the details. This proof is contained, along with all the other proofs, in the On-line Appendix.

[25]This is demonstrated in the formal proof of the Proposition.

It is important to note that the steady state stock of houses in the two communities is not tied down by Proposition 1. It tells us only that the fraction of large houses in each community must be the same.[26] The communities can be of different sizes in long run equilibrium.[27]

Finally, note that the impossibility of an asymmetric steady state implied by Proposition 1 contrasts with the conclusions of standard Tiebout models with property taxation (for example, Epple, Filimon, and Romer 1984, 1993) in which equilibrium involves communities stratified by income levels. Stratification could arise in this model if the higher θ households also preferred higher public service levels as discussed in Section 3.2. Then, under appropriate conditions, there would exist a steady state in which one community had mostly large houses and a high public service level and the other community had small houses and a low public service level. Low θ households would live in the small house community and high θ households in the community with mostly large houses. Households in the small house community would not be attracted to the large house community despite the lower tax price of services because they would not wish to spend so much on services.

4.4 Existence of equilibrium and convergence to steady states

Proposition 1 assumes an equilibrium exists and tells us what equilibrium steady states must look like. It tells us nothing about the existence of equilibrium or equilibrium steady states. Nor does it tell us whether in equilibrium the housing stock must converge to a steady state. We now discuss these issues.

Discussing convergence requires some additional terminology. For any initial state O, define the sequence of housing stocks $\langle O_t(O)\rangle_{t=0}^{\infty}$ inductively as follows: $O_0(O) = O$ and $O_{t+1}(O) = (1-d)O_t(O) + N(O_t(O))$. Intuitively, if we start in period 0 with housing stock O, at the beginning of period t the stock will be $O_t(O)$. Then, we say that the sequence of housing stocks $\langle O_t(O)\rangle_{t=0}^{\infty}$ converges to the steady state O^* if $\lim_{t\to\infty} O_t(O) = O^*$.

It is straightforward to find equilibria in which the housing stock converges to a steady state. The first task is to find a steady state. If there exists λ^* greater than 1/2 satisfying the equation

$$\lambda^* = 1 - F((1 - \delta(1-d) + \frac{cg_L^*}{\lambda^* C_L + (1-\lambda^*)C_S})(C_L - C_S)), \tag{19}$$

[26] In his static model with two housing types and a proportional property tax, Hamilton (1976) conjectured that in long run equilibrium it must be the case that the proportionate mix of housing in each community is the same. Like us, he assumed that households only differed in their demand for housing.

[27] The aggregate stock of large old houses $O_{L1}^* + O_{L2}^*$ must equal $(1-d)\lambda^*$ and the aggregate stock of small old houses $O_{S1}^* + O_{S2}^*$ must equal $(1-d)(1-\lambda^*)$. This would also be the steady state allocation with a single community so that, without zoning, there is no benefit of having two communities in this environment.

then there exists a steady state in which the fraction of large houses in each community is λ^* and the public service level is g_L^*. Similarly, if there exists λ^* less than 1/2 satisfying the equation

$$\lambda^* = 1 - F((1 - \delta(1-d) + \frac{cg_S^*}{\lambda^* C_L + (1-\lambda^*)C_S})(C_L - C_S)), \tag{20}$$

there exists a steady state in which the fraction of large houses in each community is λ^* and the public service level is g_S^*. It is straightforward to show that there must exist either a λ^* greater than 1/2 satisfying (19) or a λ^* less than 1/2 satisfying (20).[28] Indeed, both could be true. For if small home owners are choosing services, property taxes will typically be higher than if large home owners are choosing. All else equal, higher property taxes lead less households to choose large houses. It is perfectly possible, therefore, to have one steady state in which large home owners are a majority and choose low taxes, and another in which small home owners are a majority and choose high taxes.

Having found a steady state, the next step is to construct an equilibrium in which the housing stock converges to this steady state. To illustrate this concretely it is helpful to work with a specific example of the model. To this end, we set the public service benefit function $B(g)$ equal to $b \cdot g^\eta/\eta$ and assume the following parameter values:

δ	d	C_L	μ	C_S	c	η	b	$F(\theta)$
0.95	0.02	13.991	0.94	10	1	-2.33	0.2177	$uniform(0,1)$

The selection of 0.02 for the house destruction rate (d) is motivated by the work of Glaeser and Gyourko (2005), while the choice of 0.94 for the probability of remaining in the area (μ) is based on U.S. census data looking at migration flows in and out of counties.[29] The value of -2.33 for the benefit function parameter η implies a tax price elasticity of demand for public services of 0.3 which is consistent with Bergstrom and Goodman (1973). The choices made for the price of services c and the price of small homes C_S are normalizations. The parameters b and C_L are chosen so that there exists a unique steady state in which the fraction of large houses λ^* equals 0.525 and the tax rate is 0.05. This implies that 5% of average property values is spent on local services which is roughly consistent with U.S. local government spending.[30]

[28] A proof is provided in the On-line Appendix.

[29] The website link is http://flowsmapper.geo.census.gov/flowsmapper/map.html. The data come from the period 2006-2010. The website was accessed during August 2013.

[30] According to the US Census Bureau, the average price of homes sold in the US in 2010 was $272,900. The average per household spending of local governments was $14,179, which is slightly more than 5% of average home values.

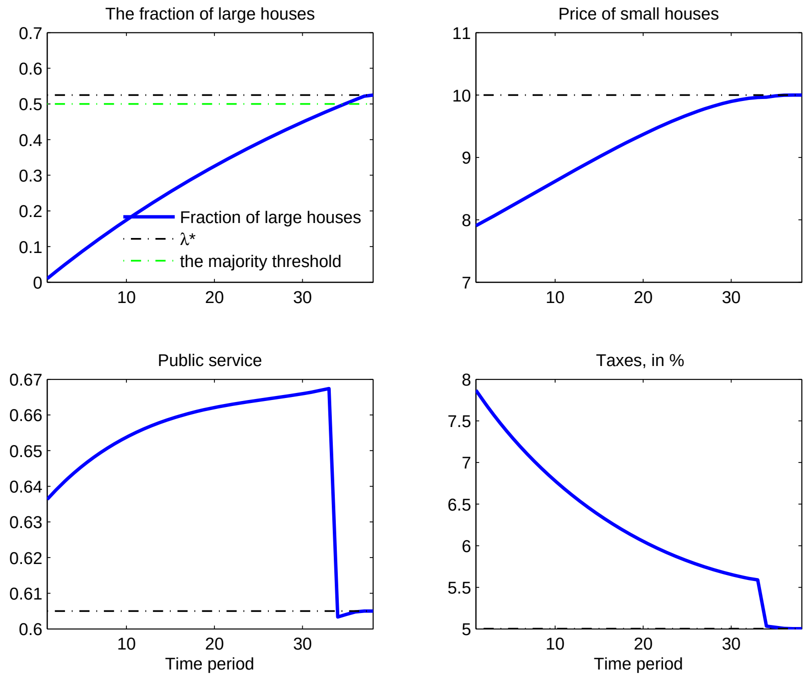

Figure 1: Convergence to Steady State in Scenario 1

The way in which the housing stock converges to the steady state in our constructed equilibrium depends on the initial condition.[31] We consider two different scenarios for illustrative purposes. In our first scenario, the initial state features a symmetric allocation of houses and the fraction of large houses is small. Figure 1 illustrates this situation. In all panels, time is measured on the horizontal axis and the equilibrium variable of interest on the vertical. Initially, all new construction will be in the form of large houses and will be balanced across the communities. The fraction of large houses increases as illustrated in the first panel. Once the fraction λ^* can be reached with new construction, construction of small homes will begin and the steady state will be reached. During the adjustment to steady state the price of small homes will be less than C_S, but it will be increasing over time as the relative fraction of large homes increases (see the second panel). The public service level provided in each community first gradually increases and then dips sharply as illustrated in the third panel. The increase reflects the fact that the improving housing stock reduces the tax price of services for small home owners. The dip arises when the community shifts from small to large home owner control. The latter face a higher tax price and thus demand a lower level of services. As illustrated in the fourth panel, the tax rate decreases over time with a sharp dip at the point at which the fraction of large houses crosses the majority threshold.

[31] A detailed description of how we compute our equilibrium can be found in the On-line Appendix.

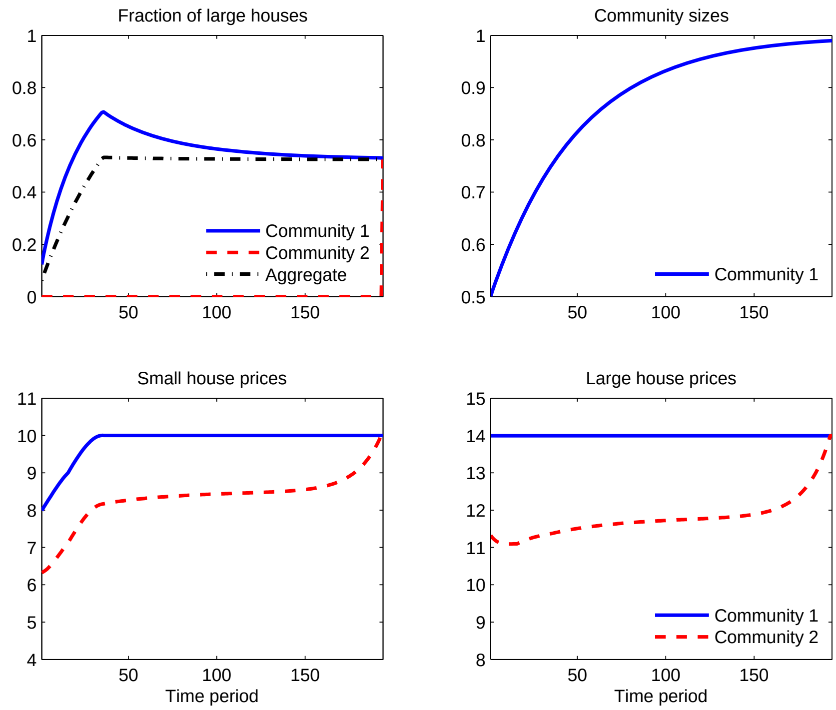

Figure 2: Convergence to Steady State in Scenario 2

In our second scenario, one community (community 1) has a larger fraction of large homes than the other, but both have less than the steady state. This situation is illustrated in Figure 2. In this case, all new construction of large homes will occur in community 1. This is because of its more favorable tax base. As illustrated in the first panel, in order to get the aggregate fraction of large houses to the required level, the fraction in community 1 must rise well above the steady state level $\lambda^* = 0.525$. When small house construction begins, it also occurs in community 1 so that this community gets bigger and bigger (see the second panel). Eventually, community 2 will become so small that one period's new construction of large homes will be sufficient to equate the fraction of large homes in the two communities. After that, new construction of both types of homes in the two communities can start. Community 2 can remain a small size or increase in size relative to community 1: this is immaterial. During the adjustment to steady state, the price of both large and small homes in community 2 will be lower than in community 1 (see the third and fourth panels). The superior tax base will therefore be capitalized into housing prices.

4.5 Efficiency

The steady states described in Proposition 1 are not efficient.[32] From an efficiency perspective, households should own a large house if their preference θ exceeds the additional resource cost of a large house in per-period terms which is

$$\theta^e \equiv (1 - \delta(1 - d))(C_L - C_S). \tag{21}$$

From (17) and (18) of Proposition 1, the steady state fraction of large houses will be too low.[33] Property taxation means that owners of large houses face a higher tax price of services and this encourages households to purchase cheaper homes. In our example, the efficient fraction of large homes is 0.725 which is significantly larger than the equilibrium fraction 0.525.

Public service provision is also inefficient. The efficient level of public services is

$$g^e \equiv g^*(c) = \arg\max \{B(g) - cg\}. \tag{22}$$

From Proposition 1, public services will be under-provided if large home owners are in the majority and over-provided if small home owners are in the majority. This reflects the fact that property taxation drives the tax price of services below the true price for small home owners and above it for large home owners. In our example, large home owners are in the majority. Thus, the equilibrium level 0.606 is smaller than the efficient level 0.633.[34]

Less obviously, when there are multiple equilibrium steady states, they may be Pareto ranked. Suppose there exists two equilibrium steady states, λ_a^* and λ_b^*, one in which large home owners form a majority and the other in which small home owners are in the majority. All households are better off when large home owners are a majority if small home owners are better off. This requires that the public service surplus enjoyed by small home owners is higher when large home owners form the majority. This is possible because, even though small home owners are not obtaining their ideal service level when large home owners are a majority, they benefit from a larger tax base.[35]

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[32] By an efficient steady state we mean one which maximizes aggregate surplus. This is the standard notion of efficiency in the literature.

[33] This is consistent with the arguments in Hamilton (1976).

[34] It is of course possible to choose the parameters b and C_L so that a majority of residents own small homes and services are over-provided at the steady state.

[35] Interestingly, when there do exist multiple Pareto ranked equilibria, citizens could be better off with a property tax limit which guaranteed that the inefficient high tax steady state could not be reached.

5 Equilibrium with exogenous zoning

To illustrate Hamilton's argument in our dynamic model, we now suppose that from period 0 onwards, one community enforces a zoning requirement that requires all newly constructed houses be large.[36] Introducing zoning in this way does not substantially complicate the definition of equilibrium. We only have to recognize that the zoning requirement impacts the housing market equilibrium by limiting the supply of small homes in the zoned community. In particular, the supply of small houses in any period is perfectly inelastic and just equals the stock that is not destroyed.[37] This means that the price of small houses in the zoned community can exceed the cost C_S. An equilibrium with exogenous zoning is defined to be an equilibrium that recognizes this constraint.

It is straightforward to characterize steady states with exogenous zoning.

Proposition 2 *In a steady state of an equilibrium with exogenous zoning, all houses in the zoned community are large and all houses in the unzoned community are small. The public service level in each community is the efficient level g^e defined in (22) and each household pays cg^e in property taxes. Households live in the zoned community only if their preference exceeds the efficient cut-off θ^e defined in (21).*

The key point to note about Proposition 2 is that the allocation of both housing and public services in the steady state is efficient. The zoned community has only large houses and the unzoned community has only small houses. Large houses are not built in the unzoned community because large home owners enjoy a lower tax price of services in the zoned community. Construction firms would love to build small houses in the zoned community so that owners could benefit from a lower tax price of services, but this is not permitted. The homogeneity of communities implies that in equilibrium the tax price faced by voters equals the true price and services are provided efficiently. Property taxes are benefit taxes in that each household's tax bill exactly equals the cost of the services it consumes. Accordingly, when choosing between the two communities, households simply decide whether the extra cost of a large house is worth it to them. This leads to efficient housing decisions.

[36] In reality, zoning requirements must be backed by some type of (non-fiscal) externality justification or they may be challenged in court. Accordingly, zoning requirements are specified in terms of quantity constraints like minimum lot size rather than simply minimum construction costs. There has been debate in the literature about how precisely such quantity constraints allow communities to regulate new construction. Fischel (1992) argues persuasively for the view that communities are able to regulate very precisely.

[37] Our analysis makes no distinction between construction which takes place on a virgin lot and that using a lot on which there was previously a house. Either type of construction must conform with zoning regulations. In reality, zoning laws often permit the rebuilding of a destroyed property even when that property is non-conforming. Thus, if a small house were destroyed, building another in its footprint would be permitted. We avoid considering such exemptions, because they would clearly prevent zoning doing the job Hamilton envisaged.

In contrast to the case without zoning, the steady state stocks of housing are uniquely defined: there are $1-F(\theta^e)$ large houses in the zoned community and $F(\theta^e)$ old small houses in the unzoned community. As in the case with no zoning, it is straightforward to construct an equilibrium in which the housing stock converges to the steady state. However, in contrast to the case with no zoning, convergence to the steady state is asymptotic. This is because if, say, small houses are present in the zoned community initially, some fraction will remain even in the very long run.

Figure 3 illustrates the transition from the steady state with no zoning to the steady state with exogenous zoning for our example. Community 1 is the community with zoning. For convenience, we assume that the two communities are initially equally sized. In the steady state without zoning, there are too few large houses because of the distortion (0.525 vs 0.725). Initially, therefore, the bulk of new construction is in the form of large houses. Since new construction of large homes takes place in community 1, this leads this community to increase in size (see the second panel). Once the supply of large homes has increased sufficiently, small house construction picks up in community 2 and the relative sizes of the communities stabilize.[38] Housing prices are significantly lower in the unzoned community (community 2) during the transition (see the third and fourth panels). Moreover, the price of small homes in community 1 is driven significantly above the replacement cost. Also notable are the impacts of zoning on public service levels in the transition. With only small houses being built in community 2, small home owners quickly become a majority and this leads to a hike in the service level in community 2 above the efficient level of 0.633 (see the sixth panel). After this point, service levels come down as the tax price for small home owners increase (see the eighth panel). By contrast, service levels increase over time in community 1 as the increasing share of large homes reduces the tax price of services (see sixth and seventh panels). The overall impact of zoning on housing prices, taxes, and tax prices is clearly beneficial for the zoned community (i.e., community 1) and adverse for the unzoned community.[39]

[38]While this is barely perceptible from the Figure, the price of small homes in community 2 starts out marginally below replacement cost and there is no new construction of small homes in community 2. New construction of small homes starts in period 5. The share of new construction devoted to large homes in community 1 is decreasing over the transition except from period 10 to 11. This reflects the fact that in period 10, the majority of home owners in community 2 become small home owners and this provides a temporary boost for small home construction. This creates the slight kink in the curve in the second panel.

[39]The model suggests a positive relationship between zoning and housing values and a negative relationship between zoning and tax rates.

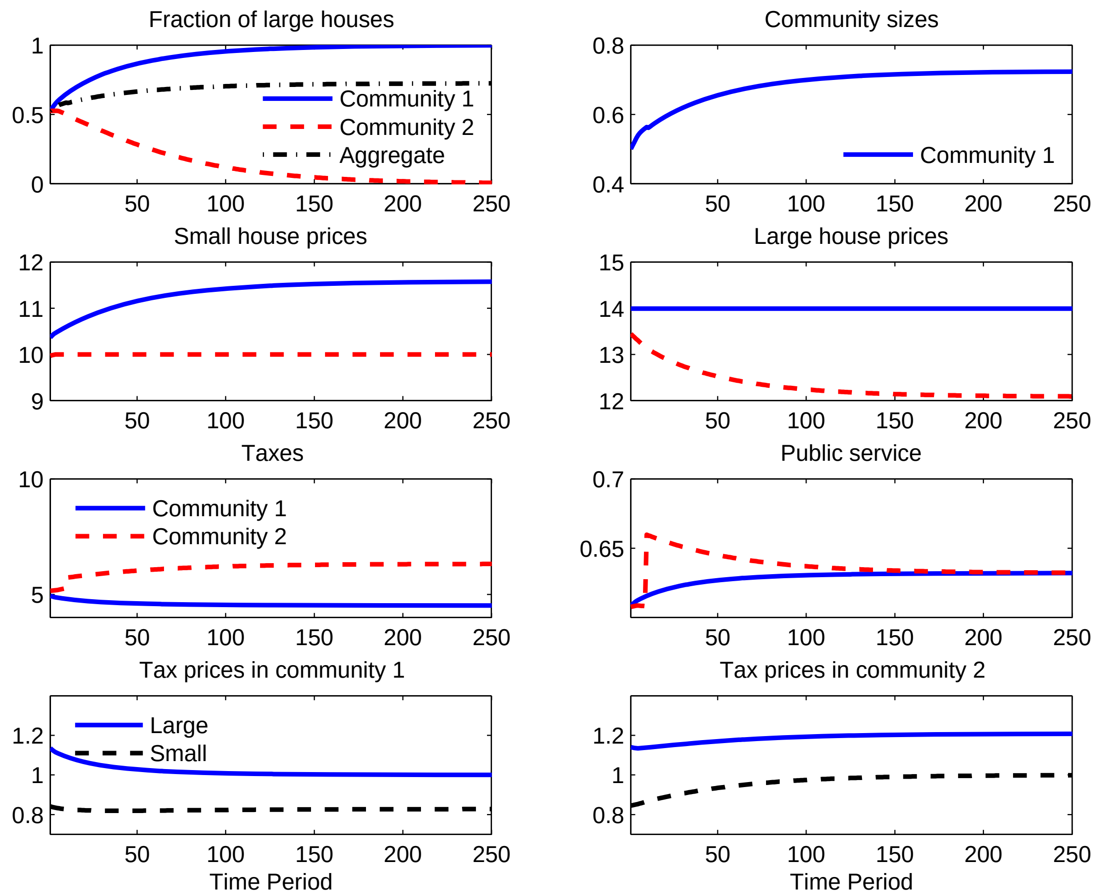

Figure 3: No Zoning to Exogenous Zoning

6 Equilibrium with endogenous zoning

The previous section showed that if one community had a zoning ordinance requiring that all newly constructed houses be large, then there is a unique steady state and it is efficient. Moreover, there exist equilibria in which housing stocks converge to this efficient steady state. However, this begs the question of whether communities would actually choose to implement zoning in this way. Specifically, if in each period the communities could choose whether to implement zoning, would the efficient outcome arise?

To analyze this question, suppose that at the very beginning of each period, before nature determines which houses are destroyed and who has to leave the area, the existing residents of each community vote whether to impose a zoning ordinance which requires that all new construction in that period be large houses.[40] Since the vote takes place before housing

[40] In some New England towns changes in zoning rules must be approved by the majority of citizens at town meetings (Gyourko, Saiz and Summers (2008)). However, more generally, zoning decisions are made by

stock is destroyed and before residents know if they have to leave, at the time of voting, all residents are in the same situation - owning homes and facing the same probability of having to leave. Let $Z_i \in \{0, 1\}$ denote community i's zoning regulation, with $Z_i = 1$ meaning the ordinance is imposed.

With endogenous zoning, the housing market equilibrium in any period depends both on the stock of houses O and the zoning regulations $Z = (Z_1, Z_2)$ chosen by residents. The housing market determines prices and new construction under these regulations. As before, after the housing market has cleared, the new residents choose service levels.

6.1 Definition of equilibrium

We begin by clarifying what is meant by equilibrium in the extended model. Let $Z(O)$ denote the zoning regulations chosen by the residents in equilibrium when the initial stock of housing is O. When deciding whether or not to implement zoning, residents must anticipate how zoning will impact housing prices and new construction. Thus, they must understand how non-equilibrium choices will impact market outcomes. We therefore let $P(O, Z)$ and $N(O, Z)$ denote housing prices and new construction with initial stock O and arbitrary (i.e., possibly non-equilibrium) zoning regulations Z. Housing market outcomes under the equilibrium regulations are denoted by $P(O)$ and $N(O)$; that is, $P(O) \equiv P(O, Z(O))$ and $N(O) \equiv N(O, Z(O))$. Through their impact on the housing market, zoning regulations will also impact service levels and tax rates. We therefore denote these by $(g_1(O, Z), g_2(O, Z))$ and $(\tau_1(O, Z), \tau_2(O, Z))$. Again, we let $(g_1(O), g_2(O))$ and $(\tau_1(O), \tau_2(O))$ denote the policies when zoning regulations are at their equilibrium levels $Z(O)$.

Households' decision-making with respect to home ownership is not fundamentally altered by the presence of zoning. With initial stock O and zoning regulations Z, the optimal house type for a household of type θ solves the problem

$$\max_{a \in \{L1, S1, L2, S2\}} \left\{ \begin{array}{c} y + \theta I_{H(a)} + B(g_{i(a)}(O, Z)) - \tau_{i(a)}(O, Z) P_a(O, Z) - P_a(O, Z) \\ +\delta[(1-d)P_a((1-d)O + N(O, Z)) + \mu V_\theta((1-d)O + N(O, Z)) + (1-\mu)\frac{y}{1-\delta}] \end{array} \right\}, \tag{23}$$

a combination of local politicians and administrative boards (typically designated the Planning and Zoning Commission and the Board of Zoning Adjustment) rather than determined by direct democracy. These boards are composed of local citizens appointed by local politicians. Our assumption is that this decision-making process will produce decisions that will reflect the will of the majority of residents. The mechanism we have in mind is that the electoral process will select local politicians whose preferences are congruent with those of the majority of voters and they will in turn appoint like-minded board members. Readers who object to this assumption can take comfort in the fact that there will often be no significant disagreement among residents. See Fischel (2001b) for more discussion of local government decision making and a defense of the median voter assumption.

where, as in the previous section, $V_\theta((1-d)O + N(O,Z))$ denotes the equilibrium payoff of the household at the beginning of a period in which it has to live in the area, does not own a house, and the housing stock is $(1-d)O + N(O,Z)$. Let $V_\theta(O,Z)$ denote the maximized value of this problem and let $\alpha_\theta(O,Z)$ denote the set of solutions; i.e., the optimal house types. Zoning impacts the housing market equilibrium by limiting the supply of small homes as discussed in the previous section. Let $\xi_{Hi}(\theta, O, Z)$ be the fraction of type θ households selecting houses of type H in community i. Again, let $\xi_{Hi}(\theta, O)$ denote the fraction under the equilibrium zoning regulations.

The additional work in defining equilibrium comes in modelling the zoning decision. Suppose the stock of houses is O and consider the zoning decision of community i. If the zoning decisions are Z, a type θ resident with a type H house in community i obtains an expected payoff

$$(1-d)P_{Hi}(O,Z) + \mu V_\theta(O,Z) + (1-\mu)\frac{y}{1-\delta}. \tag{24}$$

If community $-i$ is expected to make zoning decision Z_{-i}, it follows that a type θ resident with a type H house in community i will support imposing zoning if

$$(1-d)P_{Hi}(O,(1,Z_{-i})) + \mu V_\theta(O,(1,Z_{-i})) \geq (1-d)P_{Hi}(O,(0,Z_{-i})) + \mu V_\theta(O,(0,Z_{-i})). \tag{25}$$

If $Z(O)$ are equilibrium zoning decisions, then, for each community, $Z_i(O) = 1$ if and only if a majority of the community's residents favor zoning given that the other community is choosing $Z_{-i}(O)$.

There is a minor technicality that arises because of the indeterminacy in the allocation of household types across communities. Households with different preference types could have different preferences over zoning even if they live in the same community and own the same type of house. This is because the zoning decision could result in households making different housing choices in the future. Thus, the distribution of household preference types over the two communities will potentially matter for zoning choices. In equilibrium, at the beginning of some period t in which the initial stocks are O_t, the fraction of households of type θ owning houses of type H in community i is $\xi_{Hi}(\theta, O_{t-1})$. As was noted in Section 4.2, in equilibrium, all households buying the same type of house are indifferent between communities and hence there is an indeterminacy in how types are allocated across communities. It will simplify matters to assume that for any given stock O_t the way in which types are allocated across house types is independent of O_{t-1}. Thus, if there are two period $t-1$ stocks O_{t-1} and $\widetilde{O}_{t-1}$ which generate O_t in the sense that both $(1-d)O_{t-1} + N(O_{t-1})$ and $(1-d)\widetilde{O}_{t-1} + N(\widetilde{O}_{t-1})$ equal O_t, then for each house type Hi and every preference type θ, we require that $\xi_{Hi}(\theta, O_{t-1})$ equals $\xi_{Hi}(\theta, \widetilde{O}_{t-1})$. This seemingly harmless restriction means that associated with any initial stock O_t there will be a unique distribution of household preference types which will be independent of O_{t-1}. Without this assumption, it would be necessary to specify the distribution of types as an additional state variable which would substantially clutter notation.

We can now define equilibrium.[41] Following the definition of equilibrium without zoning, an equilibrium with endogenous zoning consists of a zoning rule $Z(O)$, a price rule $P(O,Z)$, a new construction rule $N(O,Z)$, public service rules $(g_1(O,Z), g_2(O,Z))$, tax rules $(\tau_1(O,Z), \tau_2(O,Z))$, and, for each type of household, value functions $V_\theta(O,Z)$ and $V_\theta(O)$, housing demand correspondences $\alpha_\theta(O,Z)$, and housing selection functions $\xi(\theta,O,Z)$, satisfying household optimization, housing market equilibrium, and majority rule. The housing selection functions are assumed to satisfy the assumption discussed above. The notion of housing market equilibrium defined earlier is amended to include the requirement that $N_{Si}(O,Z)$ is positive only if $P_{Si}(O,Z) = C_S$ and $Z_i = 0$. In addition, the notion of majority rule is extended to include the requirement that each community's zoning rule have the support of the majority of its residents at the beginning of the period.

6.2 Understanding household preferences over zoning

To get a feel for household preferences over zoning, consider when a type θ household with a type H house in community i will favor imposing zoning. Assume first that it will remain optimal for the household to live in a type H house that period regardless of the community's regulation decision. In equilibrium, the household will be indifferent between communities so there is no loss of generality in assuming that the household continues to locate in community i. Thus, using (23), we have that

$$V_\theta(O,(1,Z_{-i})) - V_\theta(O,(0,Z_{-i})) = B(g_i^1) - \rho_{Hi}^1 g_i^1 - P_{Hi}^1 - \left(B(g_i^0) - \rho_{Hi}^0 g_i^0 - P_{Hi}^0\right) + \delta\Delta, \quad (26)$$

where the 1 and 0 superscripts denote variables with and without zoning and Δ denotes the difference in continuation payoffs with and without zoning. Substituting (26) into (25), we can rewrite condition (25) as

$$(1-\mu-d)\left[P_{Hi}^1 - P_{Hi}^0\right] + \mu[B(g_i^1) - \rho_{Hi}^1 g_i^1 - (B(g_i^0) - \rho_{Hi}^0 g_i^0)] + \mu\delta\Delta \geq 0. \quad (27)$$

[41] Our definition of equilibrium with zoning and the preceding definition of equilibrium with no zoning follow standard practice in the dynamic political economy literature. Chapter 11 of Persson and Tabellini (2000), which draws on Krusell, Quadrini, and Rios-Rull (1997), provides a detailed exposition. As they explain, the equilibrium basically corresponds to the game-theoretic concept of Markov-perfect equilibrium, except that the policies in each period depend only on the aggregate state (i.e., O) and not each individual household's state (i.e., the type of home they own, if any).

The first term of (27) is the welfare impact resulting from changing this period's price of type Hi houses. Given that $1-\mu$ exceeds d, price increases are valued by households. Intuitively, when households are more likely to leave the area than to have to replace their houses, a higher price for the house type they own is beneficial. Since zoning restricts the supply of small homes, it is natural to expect P_{Si}^1 to be at least as large as P_{Si}^0. The impact on the price of large homes is less clear as there are both supply and demand effects.

The second term of (27) is the change in this period's public service surplus resulting from imposing zoning. This change could be positive or negative. Imposing zoning may increase the fraction of large houses which will increase surplus by reducing tax prices. However, it will also impact relative housing prices. If, for example, it raises the relative price of small houses, the tax price paid by small home owners could increase. Imposing zoning may also have political consequences that impact service levels. If large home owners become a majority, service levels may be reduced, lowering surplus for small home owners.

The final term of (27), which is the difference in continuation values, can be decomposed in exactly the same way as (26) if it is the case that the household will remain in a type H house in the next period. There will be an impact arising from changing housing prices, an impact resulting from changing public service surplus, and a future impact. Repeated application of this logic reveals that a household who will remain in a type H house as long as it remains in the area will favor zoning if

$$\sum_{t=0}^{\infty} (\mu\delta)^t \left\{(1-\mu-d)\left[P_{Hit}^1 - P_{Hit}^0\right] + \mu[B(g_{it}^1) - \rho_{Hit}^1 g_{it}^1 - (B(g_{it}^0) - \rho_{Hit}^0 g_{it}^0)]\right\} \geq 0. \quad (28)$$

Here, P_{Hit}^1 denotes the price of type Hi houses t periods after the decision to impose zoning becomes effective, P_{Hit}^0 denotes the price of type Hi houses t periods after the decision not to impose zoning becomes effective, etc. While this expression is useful in clarifying how zoning impacts household welfare, evaluating its sign is complex because the consequences of today's zoning decision for future prices are unclear in general. In particular, today's zoning decision could alter the entire path of future new construction.

The preceding analysis assumes it will remain optimal for the household to live in the same type of house regardless of the community's zoning decision. However, if zoning expands the supply of large homes, it will cause some households to live in large houses who would otherwise have lived in small houses. In this case, condition (25) can be written as:

$$(1-d)\left[P_{Hi}^1 - P_{Hi}^0\right] + \mu[\theta - (P_{Li}^1 - P_{Si}^0)] + \mu[B(g_i^1) - \rho_{Li}^1 g_i^1 - (B(g_i^0) - \rho_{Si}^0 g_i^0)] + \mu\delta\Delta \geq 0. \quad (29)$$

The first term is the change in the value of the household's current house, the second term is the benefit from owning a large home less the incremental cost,[42] and the third term is the change in public service surplus. The latter will likely be negative because the household faces a higher tax price of services with a large house.

6.3 Endogenous zoning and efficiency

We now investigate whether efficient results will obtain when communities choose their zoning regulations in a decentralized manner period by period. To pose the question clearly, we first clarify terminology. Given an equilibrium with endogenous zoning, a stock of housing O^* is a steady state if new construction at O^* is such as to maintain the stock at O^*. Drawing on the discussion in Section 5, the steady state O^* is efficient if either $Z(O^*) = (1, 0)$ and O^* equals $(1 - F(\theta^e), 0, 0, F(\theta^e))$ or $Z(O^*) = (0, 1)$ and O^* equals $(0, F(\theta^e), 1 - F(\theta^e), 0)$. As in Section 4, for any initial stock O the sequence of housing stocks $\langle O_t(O)\rangle_{t=0}^{\infty}$ is defined inductively as follows: $O_0(O) = O$ and $O_{t+1}(O) = (1 - d)O_t(O) + N(O_t(O))$. The sequence of housing stocks $\langle O_t(O)\rangle_{t=0}^{\infty}$ converges to the steady state O^* if $\lim_{t\to\infty} O_t(O) = O^*$.

Given this terminology, the question of interest is do there exist equilibria with endogenous zoning in which the housing stock converges to an efficient steady state for any initial condition? We will demonstrate that there exists no equilibrium with endogenous zoning which has a steady state that is efficient and satisfies a stability property we call "strong local stability". The steady state O^* is strongly locally stable if there exists $\varepsilon > 0$ such that for any initial stock O with the property that $\|O - O^*\| < \varepsilon$, the sequence of housing stocks $\langle O_t((1 - d)O + N)\rangle_{t=0}^{\infty}$ converges to O^* for any arbitrary vector of new construction N such that $N_{L1} + N_{S1} + N_{L2} + N_{S2} = d$. Moreover, the associated zoning rules $\langle Z(O_t((1 - d)O + N))\rangle_{t=0}^{\infty}$ equal $Z(O^*)$ in all periods. Intuitively, what this means is that if we start with an initial stock of houses close to the steady state level and perturb them by allowing one period of arbitrary new construction, then housing stocks will converge back to their steady state levels. Furthermore, the one period perturbation will not disturb zoning rules from their steady state levels. The requirement is stronger than local stability which would require that there exist $\varepsilon > 0$ such that for any initial stock O with the property that $\|O - O^*\| < \varepsilon$, the sequence of housing stocks $\langle O_t(O)\rangle_{t=0}^{\infty}$ converges to O^* and would put no restriction on the path of zoning decisions. Obviously, the smaller is d, the closer the notion of strong local stability gets to the standard idea of local stability.

Proposition 3 *There exists no equilibrium with endogenous zoning which has a steady state that is both efficient and strongly locally stable.*

[42]Note that this difference in purchase prices overstates the true increase in housing cost because it neglects the benefits of owning a more valuable asset next period, which show up in Δ.

To understand the result recall that at an efficient steady state, one community consists of large homes and the other of small. Moreover, the large home community imposes zoning and the small home community does not. Suppose the small home community were to deviate and impose zoning. What would be the response? Given the large home community is already imposing zoning, the short run effect would be to restrict the supply of small houses and raise their price. There would be no detrimental effect on public service surplus for home owners in the small home community because their community already contains no large homes. The tax price for small home owners can therefore not increase. Thus, the short run impact for residents of the small home community is positive. The long run impact is more complex because it depends on how the equilibrium responds to the deviation. In principle, the deviation could set into motion a series of changes that end up causing harm to the residents of the small home community.[43] This is impossible to know without knowing the full path of equilibrium play. Nonetheless, if the efficient steady state is strongly locally stable, there will be no long run impact of the deviation on the small home community. In the period following the one in which the deviation becomes effective, zoning rules return to their steady state values, construction of small homes resumes, and prices return to C_S. All large home construction takes place in the zoned community, so that the small home community remains homogeneous. In this case, therefore, the small home community must benefit from a deviation.

Proposition 3 does not rule out the possibility that there exists an equilibrium with endogenous zoning which has a steady state that is efficient but not strongly locally stable. Nonetheless, the logic underlying the Proposition suggests that it will certainly be difficult to find equilibria with endogenous zoning which have efficient steady states. Moreover, even if such an equilibrium exists, it seems unlikely that the housing stock and zoning rules will converge to an efficient steady state.

6.4 What will happen?

Proposition 3 suggests that endogenous zoning decisions are unlikely to produce an efficient outcome. This negative finding naturally raises the question of what will happen when zoning decisions are endogenous. This is a difficult question to answer at a general theoretical level. The potentially far-reaching consequences of today's zoning decisions for future prices make

[43] For example, suppose that community 2 deviating leads community 1 to relax its zoning requirement in the next period. This may lead low θ households to build in community 1 to take advantage of the better fiscal externality. This in turn may reduce the price of small houses in community 2. This is an unlikely scenario, to be sure, because it is not clear why community 1 would wish to remove zoning. But it is conceivable that there could be long run effects on the residents of community 1 that might justify the decision.

finding equilibria with endogenous zoning very complex. Thus, we focus on the specific example we introduced in Section 4 and look for equilibria using numerical methods.[44] We find that there exists an equilibrium in which both communities always zone whatever the housing stock. Moreover, this "always-zone equilibrium" is very robust, existing for a broad range of parameters around our chosen values.[45]

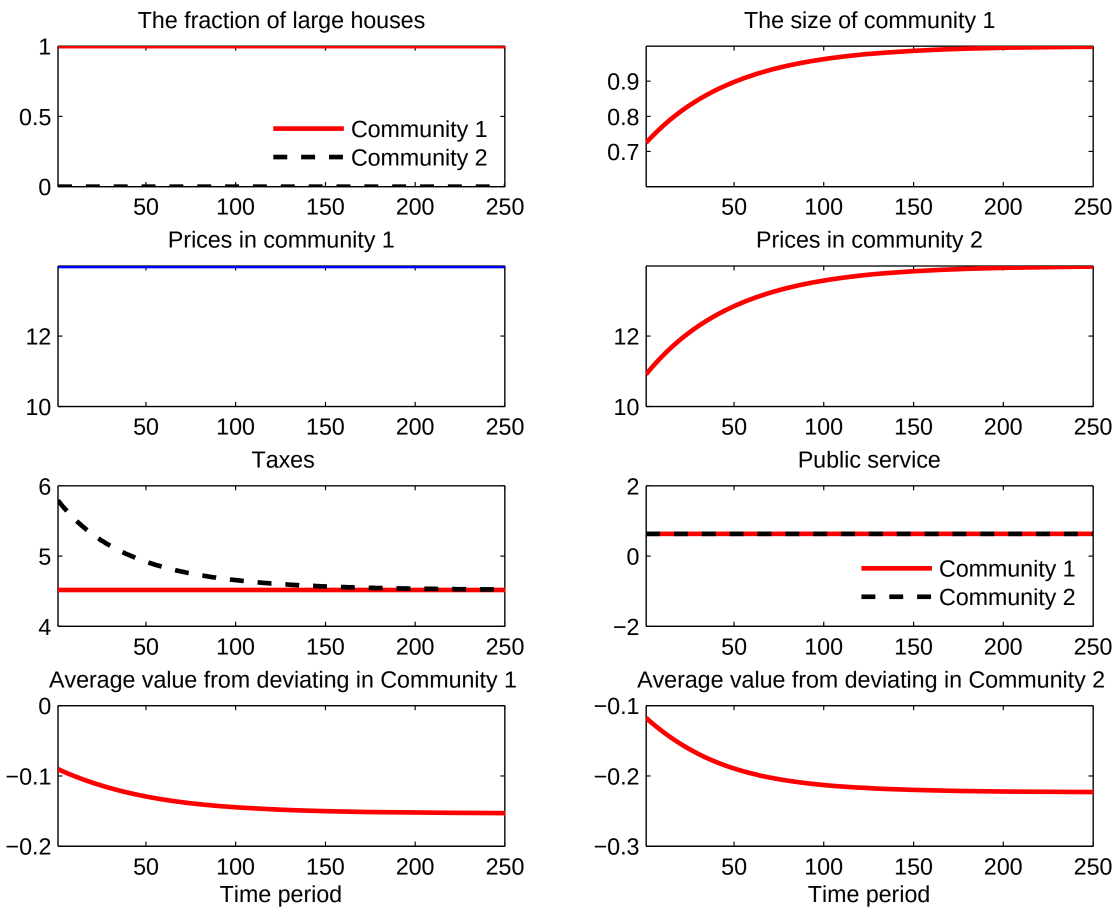

Figure 4: Exogenous Zoning to Endogenous Zoning

In our always-zone equilibrium, all houses will be large in the limit. Figure 4 illustrates what happens starting from the efficient allocation that is reached asymptotically under exogenous zoning. Community 1 is the large house community and community 2 the small

[44]It should be noted that even the static analyses of zoning in Tiebout models with property taxes by Fernandez and Rogerson (1997) and Calbrese, Epple, and Romano (2007) employ numerical methods to understand what happens in equilibrium.

[45]In particular, we have varied the parameters b and C_L so that the equilibrium with no zoning displays different steady state tax rates. The always-zone equilibrium exists for tax rates as high as 10% of average property values and as low as 1%. The always-zone equilibrium will not exist when residents have no interest in increasing their property values which is the case if $1 - \mu - d$ is negative. We have ruled out this case by our assumption that d is less than $1 - \mu$.

house community. In the always-zone equilibrium, all new construction takes place in community 1 because it has the superior tax base.[46] As a consequence of this, the size of this community converges to one (see the second panel). Community 2 remains a small house community and converges in size to zero.[47] The housing stock converges asymptotically to the allocation $O = (1, 0, 0, 0)$.[48] The price of large houses in community 1 is constant and equal to construction cost. The price of small houses in community 2 is always substantially above the replacement cost and increases over time as illustrated in the third panel. Indeed, the price of small houses converges to that of large houses in the limit. This reflects the assumption that the value of a large house θ has full support on $[0, 1]$ which means some households are basically indifferent between large and small houses. As illustrated in the sixth panel, the public service levels in the two communities remain constant at the efficient level. This reflects the fact that both communities are always homogeneous. The tax rate decreases in community 2 as the value of houses rises.

The seventh and eighth panels of Figure 4 illustrate the average loss of lifetime utility that would be experienced by the residents of the two communities if they were to deviate from equilibrium behavior by relaxing zoning for one period. They show that the cost of deviating is always well above 0 and gets higher as the stock of large houses increases. The latter partially reflects the fact that the wedge between equilibrium price and replacement cost created by zoning is increasing over time. What is not shown in the Figure is the distribution of losses across household types. While losses are decreasing for types with lower θ, support for zoning is unanimous in each community at all points in time.[49]

Figure 5 illustrates what happens under the always-zone equilibrium starting from a different initial allocation: the steady state with no zoning studied in Section 5. As discussed, the size of the two communities is not tied down in such a steady state, so we assume here

[46] It should be clear that this equilibrium relies critically on the assumption that the costs of service provision are proportional to the number of residents. This means that it does not matter to community 2's residents that it shrinks in size. Other types of agglomeration effects would also disturb this equilibrium. However, as discussed in Section 3.2, they would also directly undermine Hamilton's argument.

[47] It is important to note that community 2 never becomes empty. It always has residents and these residents vote every period to continue to zone. As a consequence, it is never possible for a low θ household to enter community 2 and build a small house. Neither is it desirable for a group of incoming households to seize political control of the community and change its zoning decision. This is because such a group must reside in the community in order to vote and residing requires purchasing a house.

[48] It should be noted that this allocation is not a steady state. This is because community 2 has no residents. Since a community with no residents cannot choose to implement zoning regulations, if housing stocks were actually at $(1, 0, 0, 0)$ there would be new construction of small homes in community 2. From the next period onward, however, there would be zoning in community 2 and housing stocks would converge asymptotically back to $(1, 0, 0, 0)$.

[49] Even when the housing stock is sufficiently dominated by large houses that low θ types live in them, removing zoning will create a large capital loss in the value of these homes and this swamps any benefit these households get from cheaper small homes.

that the two communities are initially equally sized. In our always-zone equilibrium, if the two communities are identically attractive, all new construction takes place in community 1.[50] Under this assumption, community 1 breaks away from community 2 and becomes the community in which all future new construction takes place. An initial advantage in new construction turns into a permanent advantage because of its impact on the tax base. As in the previous example, the size of community 1 converges to one and the housing stock converges asymptotically to the allocation $O = (1, 0, 0, 0)$. What is different about this particular scenario is that in the initial allocation there are a mix of houses in both commu

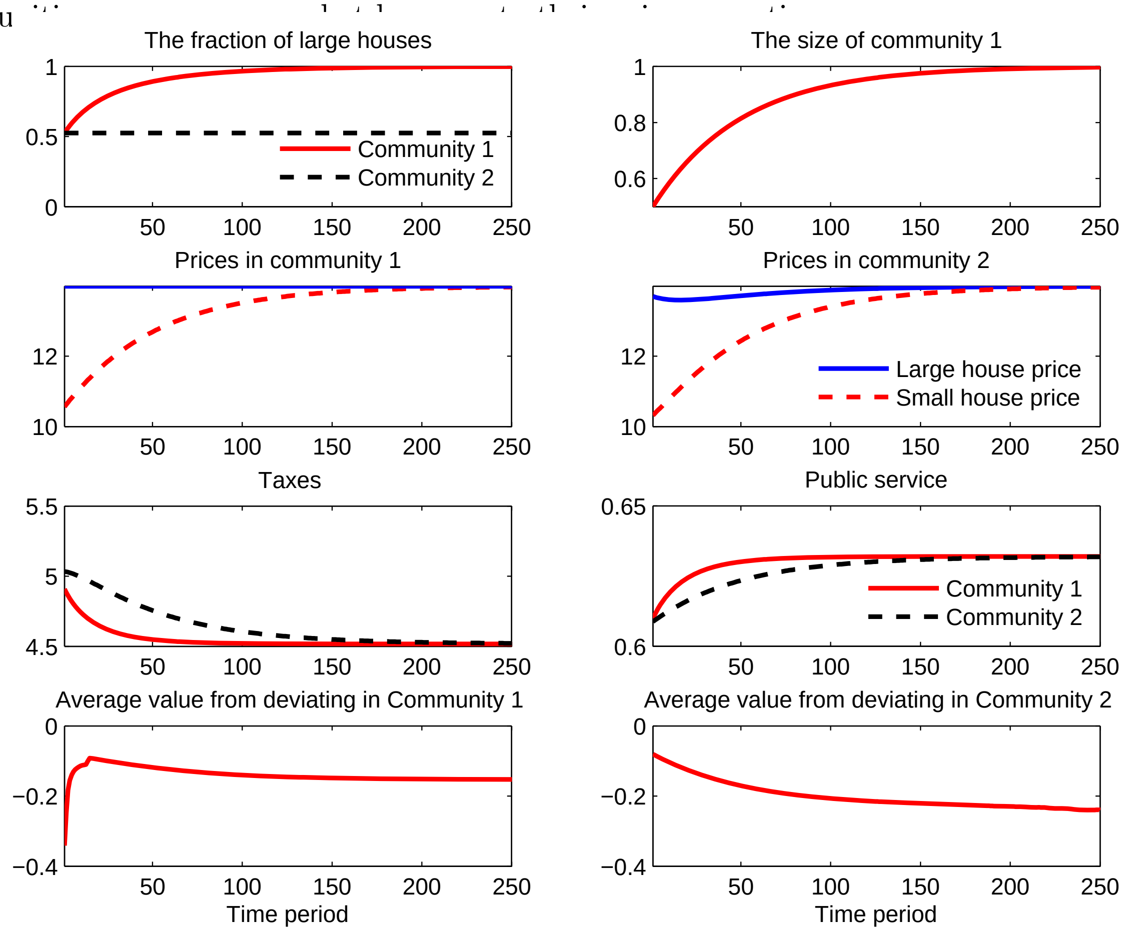

Figure 5: No Zoning to Endogenous Zoning

As illustrated in the third and fourth panels, the prices of both large and small houses are higher in community 1. The prices of small houses in both communities are increasing over

[50]This turns out to be the simplest assumption to make for computational purposes. From a theoretical perspective, we could alternatively assume that when the two communities are equally attractive, new construction is allocated between the two communities in proportion to their size. This would preserve their equal attractiveness and lead to a more balanced growth path. Under either assumption, however, all households will end up living in large houses with the efficient service level. So either type of equilibrium generates the same welfare in the long run.

time and converge to C_L in the limit. The price of large houses in community 2 initially decrease, reflecting the fact that zoning is expanding the supply of large homes and increasing the relative attractiveness of community 1. Eventually, however, these prices turn back up and also converge to C_L. As illustrated in the sixth panel, public service levels increase in both communities to the efficient level. They are initially higher in community 1 because the higher fraction of large homes create a smaller tax price for large home owners. However, tax prices and taxes converge in both communities as the price of all houses converges to C_L.

The seventh and eighth panels of Figure 5 illustrate the average loss of lifetime utility that would be experienced by the residents of the two communities if they were to deviate from equilibrium behavior by relaxing zoning for one period. They again show that the cost of deviating is always well above 0, but this cost is no longer increasing over time in community 1. It is again the case that support for zoning is unanimous in each community at all points in time.

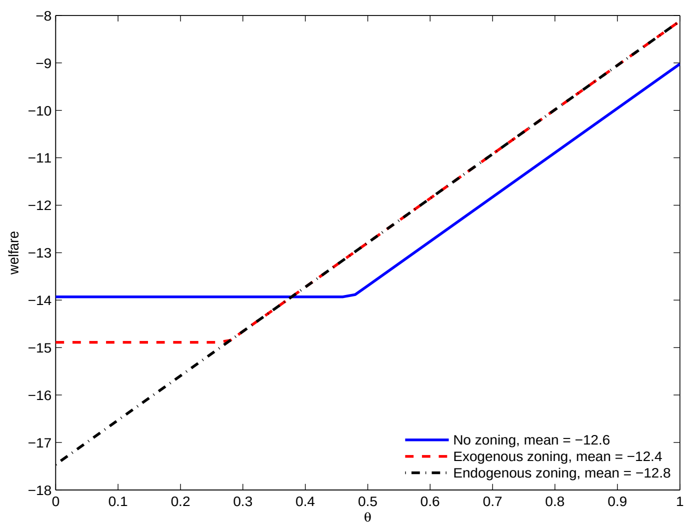

Figure 6: Welfare Comparisons

Figure 6 compares the lifetime welfare enjoyed by the different types of households in the limit allocation associated with our always-zone equilibrium with the limit allocations associated with no zoning and exogenous zoning. The Figure also documents the average welfare generated by the three regimes. The first and most important point to note from Figure 6 is that endogenous zoning generates a lower level of average welfare than does no zoning. The extreme distortions in the housing market created by over-zoning outweigh the public service distortions and more modest housing distortions arising with no zoning. The second point to note is the distributional consequences of endogenous zoning. Relative to the equilibrium with no zoning, endogenous zoning transfers welfare from those households with low preferences for large houses to those with high preferences. The intuition is clear: if a household would have bought a large house even with no zoning, he must be strictly better off with endogenous zoning because he enjoys a lower tax price of services. By contrast a household with weak housing preferences will be worse off because not only will he have to live in a bigger house than he would like, but he will also face a higher tax price for services. Indeed, such households are actually better off with no zoning than they are with exogenous zoning, precisely because they benefit from the lower tax price of services created by sharing their community with large houses.

It is worth noting that the basic distributional implications of zoning line up well with the numerical analyses of Fernandez and Rogerson (1997) and Calabrese, Epple, and Romano (2007) if we assume that θ proxies for income. The conclusions concerning aggregate welfare, however, differ from those of Calabrese et al who obtained findings concerning endogenous zoning more in line with Hamilton (1975). This discrepancy highlights the salience of the caveat they make in their conclusion: "our framing of the problem, requiring voters to abide by the constraint that is adopted, may yield more favorable efficiency effects than would emerge if voters could choose zoning to bind future residents without binding themselves" (p.43).

6.5 Implications for the Benefit View

The Benefit View of the property tax argues that, when communities are able to impose zoning ordinances, property taxes will amount to non-distortionary user charges for public services. Thus, property taxes neither distort housing choices nor public service levels. Taken literally, the view implies that property taxation has no excess burden. This justifies, for example, ignoring property taxation when trying to estimate the deadweight cost of the U.S. tax system.

The theoretical underpinnings for the Benefit View is Hamilton's argument, which is illustrated by Proposition 2. This argument presumes that households face a set of communities offering a full range of policies, begging the question of what options would be available to households in equilibrium. Proposition 3 takes issue with the idea that efficient outcomes will result when communities actually choose their zoning ordinances. It suggests that some distortions should be expected when services are financed by property taxation, even when zoning is available. This directly challenges the Benefit View.

Proposition 3 is a negative result and is silent on the precise nature of the distortions that will arise. The numerical analysis in Section 6.4 suggests that these distortions will take the form of over-consumption of housing. When parameters are set to empirically reasonable levels, there exists an equilibrium in which both communities always choose to impose zoning, and, as a result, all households live in large houses in the long run. Some fraction of these households will have willingnesses to pay for large houses smaller than their incremental cost. This is the exact opposite of the distortion in the housing market that arises without zoning and stems directly from communities' zoning decisions. On the other hand, since households live in homogeneous communities, the tax price faced by voters equals the true price. Property taxes are therefore benefit taxes in the usual sense that each household's tax bill equals the value of services it consumes. As a result, services are provided at efficient levels. This aspect of the Benefit View is therefore not challenged by our results.

7 More communities

In the environment of this paper, two communities are sufficient to achieve efficiency with exogenous zoning. Thus, it seems reasonable to study endogenous zoning with only two communities. However, this does raise the question of whether additional communities might be helpful in mitigating the problems that arise. In particular, the difficulties arise from residents using zoning to manipulate the prices of their houses. In principle, more communities should reduce communities' market power and dampen these incentives. This section considers this point by discussing the consequences of introducing a third community.

Introducing an additional community creates no real changes in the model without zoning or with exogenous zoning: Propositions 1 and 2 apply appropriately generalized. For the purposes of Proposition 2, efficiency would result if one or two communities had zoning. In the former case, there would be two small house communities and, in the latter, two large house communities. The way in which homes are allocated across the homogeneous communities in the long run is immaterial.

Proposition 3 also generalizes to the case of three communities.[51] However, the logic underlying the argument needs to be slightly modified. This is because an additional community may permit the existence of equilibria which have an efficient steady state. Consider, for example, an efficient steady state in which community 1 is zoning, communities 2 and 3 are not zoning, and all small homes are located in community 2. Suppose that at this efficient steady state, community 2 deviates by imposing zoning. This will have no impact on housing prices in community 2, since new construction of small houses will simply switch to community 3. Nonetheless, while community 2 cannot influence the price of its homes by introducing zoning at the efficient steady state, it can for housing stocks arbitrarily close

[51] The formal generalization of Proposition 3 can be found in the On-line Appendix.

to the efficient steady state. Moreover, at such housing stocks, the majority of residents of community 2 will find it beneficial to introduce zoning. It follows that the efficient steady state cannot be strongly locally stable and Proposition 3 remains true.

8 Further uses of the model

We have claimed that the dynamic Tiebout model presented in this paper could be used to analyze many issues in state and local public finance other than zoning. In this section, we briefly identify some of the issues we have in mind.

The first issue concerns the equilibrium size of communities when the average cost of providing public services as a function of population is u-shaped. The u-shaped average cost assumption was made by Tiebout (1956) who also assumed that communities with population sizes below the cost minimum would seek to expand and those with populations above the minimum would seek to contract. It is well known from static Tiebout models that communities can get stuck with an inefficient allocation of population between them. The difficulty is that individual migration decisions do not account for the associated fiscal externalities (see, for example, Buchanan and Goetz 1972). However, this ignores the possible actions that communities could take by, for example, taxing or subsidizing new construction. We could study this issue in the model of this paper by assuming that there is just one type of house and introducing the assumption that the average cost of service provision is u-shaped. To make the model a standard Tiebout model, we could also assume that households differ in their public service preferences which would create a natural division of citizens across communities. We could then allow the communities in each period to levy taxes or subsidies on new construction and study what would happen. In particular, would the system converge to an efficient allocation of citizens across communities or would such taxes and subsidies generate their own perverse incentives?

A second issue concerns the provision of local durable public goods by decentralized communities. By a durable public good is meant a public good that lasts for more than one period. Such goods include things like roads, bridges, airports, parks, and school facilities. A community's stock of durable public goods is one feature which determines its attractiveness to potential new residents. While recognized as an important issue in the literature, not much is known about the provision of local durable public goods by decentralized communities.[52] What type of distortions would we expect and how would these distortions depend on the method of financing public investments? We could study this issue in the model of this paper by assuming that there is just one type of house and introducing durable public goods as

[52] See Stiglitz (1983) for an early discussion and Conley, Driskill, and Wang (2013) for a recent analysis.

opposed to public services. It would again be natural to assume that households differ in their preferences for these public goods and to assume that the public goods were congestible. The simplest financing arrangement to study would be that all investment has to be financed by taxation of current residents. More advanced models could allow communities to issue debt to finance investment or to levy user charges on new construction. The key complication of such a model is that the state space would include not only the stocks of housing in each community but also the stocks of durable goods. With tax finance, existing residents may have an incentive to underinvest in durable public goods because they may not be around to enjoy the benefits. In addition, they may anticipate that investment would just induce an inflow of new residents. The use of debt or user charges on new construction may mitigate these difficulties, but may create their own set of distortions.

A final issue concerns the competition by communities to attract private capital to boost employment opportunities for their residents. Again, this issue is recognized as important in the literature and there is a considerable amount of work on it from a static perspective (see Wilson and Wildasin 2004 for an overview). One could consider a model in which entrepreneurs had to decide how to allocate capital across communities and the aggregate level of capital in each community along with the number of residents determined the wage rate residents received. It would be natural to assume that capital was at least partially sunk so that it could not be costlessly reallocated across communities in each period. Thus, the state variables would include both housing stocks and private capital levels in the communities. It might also be interesting in such a model to introduce shocks that alter the natural advantages/disadvantages of the communities. One could then study how communities respond to negative shocks which make them less attractive to entrepreneurs. The desire to preserve property values may lead communities to act aggressively to prevent outflows of private capital. Presumably, however, after a sufficiently negative series of shocks, efforts to maintain the community will cease and property values will collapse.

9 Conclusion

This paper has presented a new dynamic Tiebout model and used it to revisit a classic question in public finance concerning the ability of a system of local governments financing public services with property taxes to produce efficient allocations of both housing and services when they can implement zoning ordinances. The dynamic structure of the model captures aspects of zoning that seem intuitively important. In particular, zoning is chosen by existing residents and impacts new construction. Existing property is grandfathered in and thus owners are not bound by the regulations they set. They can therefore use such regulations to extract rent from future residents. While confirming Hamilton's insight that such a system can in principle yield efficient results, the analysis of the model suggests that local governments will be unlikely to choose the right zoning ordinances. Indeed, allowing local governments to implement zoning may actually reduce long run welfare.

The results challenge the Benefit View of the property tax in the sense that they contradict the argument that combining decentralized property taxation and zoning will yield first best results. However, they do not contradict the narrower position that, with zoning, property taxes will be benefit taxes. It is just that this narrower claim is besides the point. Property taxes being benefit taxes does not imply that housing decisions are undistorted, since such decisions can be directly distorted by the supply restrictions embodied in zoning.

It should nonetheless be stressed that the results of this paper do not imply that property taxation is a bad way of financing local government.[53] In particular, if we take it as an institutional constraint that communities are able to implement zoning ordinances, then property taxation may be an excellent system of finance. To illustrate, suppose that to keep housing prices high, communities would implement zoning whether they financed public services with head taxes or property taxes. Then, property taxes will be equivalent to head taxes. The point is that housing is distorted by zoning under both methods of finance and property taxes create no additional distortions relative to head taxes. Sales or income taxation, on the other hand, will distort consumption and labor supply decisions and will not obviously change housing distortions by influencing zoning decisions.

The model presented here could be developed to shed additional light on the political economy of zoning. A useful and relatively simple way to enrichen the model would be to assume that higher θ households not only valued housing more than low types but also public services. As noted earlier, this would likely permit the existence of stratified communities without zoning. It would also create additional conflict within communities concerning the optimal level of services and it would be interesting to see how this might impact communities' zoning decisions. A more challenging extension would be to introduce more sizes (or qualities) of houses, preferably a continuum. Communities would then choose a minimum housing size from an interval a la Fernandez and Rogerson (1997) and Calabrese, Epple, and Romano (2007). Under this extension, even with zoning, communities would consist of a range of housing types, rather than being homogeneous. Moreover, it would be possible to analyze the determinants of the stringency of zoning. The technical difficulty to be overcome is that the state space would consist of the entire distribution of houses in the two commu-

[53] For a nice overview of the debate about whether local governments should use property or income taxes see Oates and Schwab (2004). For alternative political economy perspectives see Glaeser (1996) and Hoxby (1999).

nities rather than just two pairs of numbers as in this paper. Another interesting extension would be to introduce the possibility of subdividing large homes into small homes.[54] With such a possibility, zoning would have to also rule-out multiple occupancy of large homes. Subdivision may make the political support for zoning more fragile because large home owners who would prefer to live in small homes would not face such a drop in their home values were zoning to be removed.

[54] We thank an anonymous referee for this suggestion.

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10 On-line Appendix

10.1 Proofs

10.1.1 Proof of Proposition 1

Suppose first that $O^*_{Hi} > 0$ for all Hi. Since O^* is an equilibrium steady state, then, $N(O^*) = dO^*$. Thus, since $O^*_{Hi} > 0$ for all Hi, it must be the case that there is new construction of both types of houses in both communities. Accordingly, housing prices must equal construction costs so that $P(O^*) = (C_L, C_S, C_L, C_S)$. It must also be the case that the fraction of large houses in each community is the same; that is, $\lambda_1(O^*) = \lambda_2(O^*) = \lambda^*$. For if one community had a greater fraction of large houses, the public service surplus enjoyed by large house owners in that community would be higher than in the other which would violate (12). Since both house prices and the fraction of large houses are the same across the two communities, it follows from (9) and (10) that service levels and taxes are also the same. If a majority of households own large houses ($\lambda^* \geq 1/2$), then, from (8) and (9), the public service level will be g^*_L and, from (13), households live in large houses only if their preference exceeds the expression in (17). If a majority of households own small houses ($\lambda^* < 1/2$), the public service level is g^*_S and households live in large houses only if their preference exceeds the expression in (18).

It remains to consider the possibility that $O^*_{Hi} = 0$ for some house type Hi. Since the stock of housing must be sufficient to accommodate those households who need to reside in the area, we know that $\sum_i \sum_H O^*_{Hi} = 1$. It cannot be the case that there are no small houses ($O^*_{S1} = O^*_{S2} = 0$) because this would imply that the equilibrium prices of small houses must exceed C_S. This reflects the assumption that the distribution of household types has full support on the interval $[0, \overline{\theta}]$. This means that there are household types who are close to indifferent between large and small houses and they would be willing to pay close to C_L for a small house. By assumption, C_L is larger than C_S.

Suppose there are no large houses ($O^*_{L1} = O^*_{L2} = 0$). Then both communities would consist of small houses implying that $\lambda_1(O^*) = \lambda_2(O^*) = 0$. The steady state price of small houses would be C_S and communities would choose public good service levels level equal to $g^*(c)$ and a tax rate of $cg^*(c)/C_S$. The steady state price of large houses would have to be less than or equal to C_L so that construction firms had no incentive to supply large homes. In order for this to be satisfied, all households must weakly prefer to live in a small house given that small and large houses are priced at replacement cost and the tax rate is

$cg^*(c)/C_S$. This requires that

$$\overline{\theta} \leq (1-\delta(1-d))(C_L - C_S) + \frac{c(C_L - C_S)}{C_S} g^*(c).$$

All we have assumed is that $\overline{\theta}$ exceeds $(1-\delta(1-d))(C_L - C_S)$, so this is possible given our assumptions. However, the existence of such a steady state is perfectly consistent with the Proposition which does not require that $\lambda_i(O^*) > 0$.

We are left with two possibilities: there are no small houses in just one community ($O^*_{Si} = 0$ for some i) and there are no large houses in just one community ($O^*_{Li} = 0$ for some i). The first possibility is easy to rule out. Suppose that community 1 were the community in which there were no small houses (i.e., $O^*_{S1} = 0$). It cannot be the case that $O^*_{L2} > 0$, because then $(P_{L1}(O^*), P_{L2}(O^*), P_{S2}(O^*))$ would equal (C_L, C_L, C_S) and large home owners would strictly prefer to buy large homes in community 1. It cannot be the case that $O^*_{L2} = 0$, because then $(P_{L1}(O^*), P_{S1}(O^*), P_{S2}(O^*))$ would equal (C_L, P_{S1}, C_S) for some $P_{S1} \leq C_S$, the public service levels in both communities would equal $g^*(c)$, and the tax rates would be $cg^*(c)/C_L$ and $cg^*(c)/C_S$ respectively. Small home owners would then strictly prefer to buy small homes in community 1.

The second possibility is harder to rule out and this is where Assumption 1 comes into play. Suppose that community 1 is the community in which there are no large houses (i.e., $O^*_{L1} = 0$). Then the steady state prices $(P_{S1}(O^*), P_{L2}(O^*), P_{S2}(O^*))$ must equal (C_S, C_L, C_S). Moreover, the public service level in community 1 must equal $g^*(c)$, and the tax rate would be $cg^*(c)/C_S$. In community 2, the tax price for small home owners would equal $cC_S/\left(C_L\lambda_2(O^*) + C_S(1-\lambda_2(O^*))\right)$ which is lower than the community 1 tax price of c. It must be the case that large home owners are in the majority in community 2 (i.e., $\lambda_2(O^*) \geq 1/2$) because otherwise small house owners would strictly prefer to buy in community 2 to benefit from the lower tax price. Given that home prices are the same, in order for small home owners to be indifferent between buying in the two communities, it must be the case that their public service surplus is the same. This requires that the fraction of large houses in community 2 $\lambda_2(O^*)$ satisfies the equation:

$$B(g^*(c)) - cg^*(c) = \left[\begin{array}{c} B(g^*(\frac{cC_L}{C_L\lambda_2(O^*)+C_S(1-\lambda_2(O^*))})) \\ -\frac{cC_S}{C_L\lambda_2(O^*)+C_S(1-\lambda_2(O^*))} g^*(\frac{cC_L}{C_L\lambda_2(O^*)+C_S(1-\lambda_2(O^*))}) \end{array} \right].$$

It follows from this discussion that a necessary condition for the existence of a steady state in which one community is all small houses and the other has a mix of houses is that there exists $\lambda \geq 1/2$ such that

$$B(g^*(c)) - cg^*(c) = B(g^*(\frac{cC_L}{C_L\lambda + C_S(1-\lambda)})) - \frac{cC_S}{C_L\lambda + C_S(1-\lambda)}g^*(\frac{cC_L}{C_L\lambda + C_S(1-\lambda)})$$

Since the right hand side of this equation is increasing in λ there will not exist an asymmetric equilibrium if

$$B(g^*(c)) - cg^*(c) < B(g^*(\frac{2cC_L}{C_L + C_S})) - \frac{2cC_S}{C_L + C_S}g^*(\frac{2cC_L}{C_L + C_S}). \tag{30}$$

Intuitively, this is saying that the surplus a small homeowner would obtain living in a community in which 1/2 of the homes are large and the public service level is chosen by large home owners, is strictly larger than that they would enjoy living in a community in which all homes are small. We will show that inequality (30) is true under Assumption 1.

Defining the function $\varphi : [C_S, C_L] \to \Re$ as follows:

$$\varphi(x) = B(g^*(\frac{2cx}{x + C_S})) - \frac{2cC_S}{x + C_S}g^*(\frac{2cx}{x + C_S}),$$

it suffices to show that $\varphi(C_L)$ exceeds $B(g^*(c)) - cg^*(c)$. Note that $\varphi(C_S)$ equals $B(g^*(c)) - cg^*(c)$, and so inequality (30) must be true if $\varphi'(x) \geq 0$ over the relevant range. Differentiating, we have that

$$\varphi'(x) = \left(B'(g^*(\frac{2cx}{x + C_S})) - \frac{2cC_S}{x + C_S}\right)\frac{dg^*}{dx} + \frac{2cC_S}{(x + C_S)^2}g^*(\frac{2cx}{x + C_S}).$$

Using the fact that for any tax price ρ, we have that $B'(g^*(\rho)) = \rho$, we can write this as

$$\varphi'(x) = \left(\frac{2c(x - C_S)}{x + C_S}\right)\frac{dg^*}{dx} + \frac{2cC_S}{(x + C_S)^2}g^*(\frac{2cx}{x + C_S}).$$

Noting that

$$\frac{dg^*}{dx} = \frac{dg^*(\frac{2cx}{x+C_S})}{d\rho}\frac{d\rho}{dx} = \frac{dg^*(\frac{2cx}{x+C_S})}{d\rho}\left(\frac{2cC_S}{(x + C_S)^2}\right),$$

we can write

$$\varphi'(x) = \frac{2cC_S}{(x + C_S)^2}\left[\left(\frac{2c(x - C_S)}{x + C_S}\right)\frac{dg^*(\frac{2cx}{x+C_S})}{d\rho} + g^*(\frac{2cx}{x + C_S})\right].$$

It follows that inequality (30) must be true if for all $x \in [C_S, C_L]$

$$g^*(\frac{2cx}{x+C_S}) \geq -\left(\frac{2c(x-C_S)}{x+C_S}\right)\frac{dg^*(\frac{2cx}{x+C_S})}{d\rho}.$$

Recalling that $\varepsilon_d(\rho)$ denotes the elasticity of demand for public services at tax price ρ, this is equivalent to

$$\varepsilon_d(\frac{2cx}{x+C_S}) \leq \frac{1}{1-\frac{C_S}{x}}.$$

This follows immediately from Assumption 1. ■

10.1.2 Existence of a steady state with no zoning

In Section 4.4, we claim that there will exist a steady state with no zoning. Given the discussion in the text, this will be true if either there exists λ^* greater than 1/2 satisfying the equation

$$\lambda^* = 1 - F((1-\delta(1-d) + \frac{cg_L^*}{\lambda^* C_L + (1-\lambda^*)C_S})(C_L - C_S)),$$

or there exists λ^* less than 1/2 satisfying the equation

$$\lambda^* = 1 - F((1-\delta(1-d) + \frac{cg_S^*}{\lambda^* C_L + (1-\lambda^*)C_S})(C_L - C_S)).$$

In the former case, there exists a steady state in which the fraction of large houses in each community is λ^* and the public service level is g_L^* and in the latter there exists a steady state in which the fraction of large houses in each community is λ^* and the public service level is g_S^*.

Define the function $\varphi(\lambda)$ on the interval $[0,1]$ as follows:

$$\varphi(\lambda) = \begin{cases} 1 - F((1-\delta(1-d) + \frac{cg^*(\frac{cC_S}{C_L\lambda + C_S(1-\lambda)})}{\lambda C_L + (1-\lambda)C_S})(C_L - C_S)) & \text{if } \lambda \in [0, 1/2) \\ 1 - F((1-\delta(1-d) + \frac{cg^*(\frac{cC_L}{C_L\lambda + C_S(1-\lambda)})}{\lambda C_L + (1-\lambda)C_S})(C_L - C_S)) & \text{if } \lambda \in [1/2, 1] \end{cases}.$$

Then it is enough to show that there exists a solution to the equation $\lambda^* = \varphi(\lambda^*)$. Consider the behavior of the function $\varphi(\lambda)$ on the interval $[0,1]$. First, note that because $cg^*(c)/C_S$ exceeds $cg^*(c)/C_L$, we have that $\varphi(0) \leq \varphi(1)$. Second, note that we can assume without loss of generality that

$$(1-\delta(1-d) + \frac{cg^*(c)}{C_S})(C_L - C_S) < \overline{\theta},$$

for if this inequality is not satisfied then $\lambda^* = 0$ is a solution. This assumption implies that $\varphi(0) > 0$. Third, since $(1 - \delta(1-d) + \frac{cg^*(c)}{C_L})(C_L - C_S)$ is positive, we have that $\varphi(1) < 1$. Fourth, given the properties of the public service benefit function $B(g)$, the optimal public service level $g^*(\rho)$ is a continuous function of the tax price. This implies that the function $\varphi(\lambda)$ is continuous on $[0, 1/2)$ and continuous on $[1/2, 1]$. Finally, given that $g^*(\rho)$ is decreasing in ρ, we have that

$$\begin{aligned}\lim_{\lambda \nearrow 1/2} \varphi(\lambda) &= 1 - F((1 - \delta(1-d) + \frac{2cg^*(\frac{2cC_S}{C_L - C_S})}{C_L - C_S})(C_L - C_S)) \\ &\leq 1 - F((1 - \delta(1-d) + \frac{2cg^*(\frac{2cC_L}{C_L - C_S})}{C_L - C_S})(C_L - C_S)) = \varphi(1/2)\end{aligned}$$

With this understanding of the behavior of the function $\varphi(\lambda)$, we can now prove that there must exist a solution to the equation $\lambda^* = \varphi(\lambda^*)$. Suppose there does not exist a solution on the interval $[0, 1/2)$. Then we know that since $\varphi(0) > 0$ and $\varphi(\lambda)$ is continuous on $[0, 1/2)$, it must be the case that $\varphi(\lambda) > \lambda$ for all λ on the interval $[0, 1/2)$. It follows that $1/2 < \lim_{\lambda \nearrow 1/2} \varphi(\lambda) \leq \varphi(1/2)$. But we know that $\varphi(1)$ is less than 1. Given that $\varphi(\lambda)$ is continuous on $[1/2, 1]$, there must exist a λ^* between 1/2 and 1 such that $\lambda^* = \varphi(\lambda^*)$. ■

10.1.3 Proof of Proposition 2

Suppose that community 1 is the zoned community. If O^* is a steady state, then, under zoning, it must be the case that $O^*_{S1} = 0$ and hence $\lambda_1(O^*) = 1$. It must also be the case that $O^*_{L2} = 0$ and hence that $\lambda_2(O^*) = 0$. To see why, suppose, to the contrary, that $O^*_{L2} > 0$. Then it must be the case that the steady state price of large houses in both communities is C_L. Since the price of small houses in community 2 is C_S, the tax price of public services is lower for large house owners in community 1. But this means public service surplus enjoyed by large house owners in community 1 is higher than in community 2 which would violate (12). Since $P_{L1}(O^*) = C_L$ and $\lambda_1(O^*) = 1$ and $P_{S2}(O^*) = C_S$ and $\lambda_2(O^*) = 0$, it follows from (9) and (10) that

$$(g_1(O^*), \tau_1(O^*)) = (g^*(c), \frac{cg^*(c)}{C_L})$$

and that

$$(g_2(O^*), \tau_2(O^*)) = (g^*(c), \frac{cg^*(c)}{C_S}).$$

Households living in community 1 pay property taxes equal to $\frac{cg^*(c)}{C_L} C_L = cg^*(c)$ and households living in community 2 pay property taxes equal to $\frac{cg^*(c)}{C_S} C_S = cg^*(c)$. From (4), it follows that a household of type θ will prefer living in a large house in community 1 to a small house in community 2 if

$$\theta + B(g^*(c)) - cg^*(c) - C_L + \delta(1-d)C_L \geq B(g^*(c)) - cg^*(c) - C_S + \delta(1-d)C_S$$

or, equivalently, if their preference θ exceeds θ^e as defined in (22). ■

10.1.4 Proof of Proposition 3

Let

$$\left\langle \begin{array}{c} Z(O), P(O,Z), N(O,Z), (g_1(O,Z), g_2(O,Z)), \\ (\tau_1(O,Z), \tau_2(O,Z)), (V_\theta(O,Z), V_\theta(O), \alpha_\theta(O,Z))_\theta, \xi(\theta,O,Z) \end{array} \right\rangle$$

be an equilibrium with endogenous zoning and let O^* be a steady state. Suppose, contrary to the Proposition, that O^* is both efficient and strongly locally stable. By definition, this implies that either $Z(O^*) = (1,0)$ and O^* equals $(1 - F(\theta^e), 0, 0, F(\theta^e))$ or $Z(O^*) = (0,1)$ and O^* equals $(0, F(\theta^e), 1 - F(\theta^e), 0)$. For concreteness, assume the former, so that the large houses are in community 1. By the definition of a steady state, this implies that $N(O^*) = d(1 - F(\theta^e), 0, 0, F(\theta^e))$.

We will show that all residents of community 2 would be better off imposing zoning if the state were O^*, which is inconsistent with the fact that $Z_2(O^*) = 0$. Recall that community 2 consists of all households of type $\theta \leq \theta^e$ and that at the time of voting they all own small houses. From (24), the equilibrium continuation payoff for a resident of type θ in community 2 is

$$(1-d)C_S + \mu V_\theta(O^*) + (1-\mu)\frac{y}{1-\delta}.$$

Since O^* is a steady state, we have that for all $\theta \leq \theta^e$

$$V_\theta(O^*) = y + B(g^e) - cg^e - C_S + \delta[(1-d)C_S + \mu V_\theta(O^*) + (1-\mu)\frac{y}{1-\delta}],$$

which implies that

$$V_\theta(O^*) = V^* \equiv \frac{y + B(g^e) - cg^e - C_S + \delta[(1-d)C_S + (1-\mu)\frac{y}{1-\delta}]}{1-\delta\mu}.$$

Note that this continuation payoff is independent of θ.

The continuation payoff for a resident of type θ in community 2 if community 2 deviates from equilibrium play and implements zoning is

$$(1-d)P_{S2}(O^*, (1,1)) + \mu V_\theta(O^*, (1,1)) + (1-\mu)\frac{y}{1-\delta}.$$

To evaluate this, we need to know what happens following community 2's deviation to impose zoning. In the period of the deviation, the initial stock of housing will be $O_0 = O^*$, new construction will be $N_0 = N(O^*, (1,1))$, housing prices will be $P_0 = P(O^*, (1,1))$, public service levels will be $(g_{10}, g_{20}) = (g_1(O^*, (1,1)), g_2(O^*, (1,1)))$, and tax rates will be $(\tau_{10}, \tau_{20}) = (\tau_1(O^*, (1,1)), \tau_2(O^*, (1,1)))$. The housing stock at the beginning of the period following the deviation will be $O_1 = (1-d)O_0 + N_0$. Define the sequence of housing stocks $\langle O_t(O_1) \rangle_{t=1}^{\infty}$ inductively as follows: $O_1(O_1)$ equals O_1 and $O_{t+1}(O_1)$ equals $(1-d)O_t(O_1) + N(O_t(O_1))$ where $N(O_t(O_1))$ is the equilibrium level of new construction associated with housing stocks $O_t(O_1)$; that is, $N(O_t(O_1))$ equals $N(O_t(O_1), Z(O_t(O_1)))$. Then, $O_t = O_t(O_1)$ is the housing stock that will prevail at the beginning of the period t periods after the deviation. In that period, zoning rules will be $Z_t = Z(O_t)$, new construction will be $N_t = N(O_t)$, housing prices will be $P_t = P(O_t)$, public service levels will be $(g_{1t}, g_{2t}) = (g_1(O_t), g_2(O_t))$, and tax rates will be $(\tau_{1t}, \tau_{2t}) = (\tau_1(O_t), \tau_2(O_t))$.

By assumption the steady state O^* is strongly locally stable. Since $\|\mathbf{O}^* - \mathbf{O}^*\| = 0$ and $N_{L10} + N_{S10} + N_{L20} + N_{S20} = d$, this implies that $\lim_{t\to\infty} O_t = O^*$ and that $Z_t = (1,0)$ for all $t \geq 1$. With this information, we can now establish three properties of the sequence of policies that follow community 2's deviation.

Property 1: For sufficiently large t, $P_{L1t} = C_L$ and $P_{S2t} = C_S$.

Proof of Property 1: To prove this it is enough to show that for sufficiently large t, $N_{L1t} > 0$ and $N_{S2t} > 0$. But this follows from the fact that $\lim_{t\to\infty} O_t = O^*$ and that $O^* = (1 - F(\theta^e), 0, 0, F(\theta^e))$. ■

Property 2: For all $t = 0, ..., \infty$, $(\lambda_{1t}, \lambda_{2t}) = (1, 0)$, where λ_{it} is the fraction of post-construction houses that are large in community i, t periods after the deviation.

Proof of Property 2: We know that $\lambda_{10} = 1$ because $(O_{L10}, O_{S10}) = (1 - F(\theta^e), 0)$ and, since zoning is in place in community 1 in the period of deviation, we have that $N_{S10} = 0$. Moreover, by strong local stability, $Z_{1t} = 1$ for all $t \geq 1$ and thus it must be that $\lambda_{1t} = 1$ for all t.

We know that $(O_{L20}, O_{S20}) = (0, F(\theta^e))$. We also claim that for all $t \geq 0$, $N_{L2t} = 0$. To see this, suppose, to the contrary, that $N_{L2t} > 0$. Then, in order for households to want to buy these houses it must be that

$$[B(g_{2t}) - \tau_{2t} C_L] - C_L + \delta(1-d)P_{L2t+1} \geq [B(g^e) - cg^e] - P_{L1t} + \delta(1-d)P_{L1t+1}.$$

But because community 2 has small houses and community 1 does not, it must be that

$$B(g_{2t}) - \tau_{2t} C_L < B(g^e) - cg^e.$$

Thus, for the above inequality to hold, we must have that $P_{L2t+1} > P_{L1t+1}$. But we know that community arbitrage implies that

$$[B(g_{2t+1}) - \tau_{2t+1}P_{L2t+1}] - P_{L2t+1} + \delta(1-d)P_{L2t+2} = [B(g^e) - cg^e] - P_{L1t+1} + \delta(1-d)P_{L1t+2}.$$

But again because community 2 has small houses, it must be that

$$[B(g_{2t+1}) - \tau_{2t+1}P_{L2t+1}] < [B(g^e) - cg^e].$$

Thus, we require $P_{L2t+2} > P_{L1t+2}$. Continuing this line of argument, we conclude that $P_{L2t} > P_{L1t}$ for all $t = 1, ..., \infty$. But we know from Property 1 that for sufficiently large t, it must be that $P_{L1t} = C_L$. It follows that for all $t \geq 0$, $\lambda_{2t} = 0$. ■

Property 3: $P_{S20} > C_S$ and for all $t \geq 1$, $P_{S2t} = C_S$.

Proof of Property 3: From Property 1 we know that for sufficiently large t it must be that $(P_{L1t}, P_{S2t}) = (C_L, C_S)$. Let $\widehat{t}$ be the largest period in which $(P_{L1t}, P_{S2t}) \neq (C_L, C_S)$. Suppose first that $\widehat{t} = 0$. Then all we need to show is that $P_{S20} > C_S$. We know that $O_0 = (1 - F(\theta^e), 0, 0, F(\theta^e))$. From Property 2, we know that $N_0 = (d, 0, 0, 0)$ and hence that $P_{L10} = C_L$. We also know that $P_{L11} = C_L$ and that $P_{S21} = C_S$. Suppose to the contrary that $P_{S20} \leq C_S$. Then, it must be that all types with preferences less than θ^e strictly prefer small houses in community 2 implying that demand is at least equal to $F(\theta^e)$. Supply, however, is equal to $(1-d)F(\theta^e)$.

Now suppose that $\widehat{t} \geq 1$. Since $Z_{\widehat{t}} = (1, 0)$, there are two possibilities in period $\widehat{t}$: (i) $P_{L1\widehat{t}} = C_L$, $P_{S2\widehat{t}} < C_S$, and $N_{L1\widehat{t}} = d$, and (ii) $P_{L1\widehat{t}} < C_L$, $P_{S2\widehat{t}} = C_S$, and $N_{S2\widehat{t}} = d$. We now show that possibility (i) cannot arise. Suppose, to the contrary, that it does arise. Then, given that $(P_{L1\widehat{t}+1}, P_{S2\widehat{t}+1}) = (C_L, C_S)$, we know that it must be that $(1-d)O_{L1\widehat{t}} + d \leq 1 - F(\theta^e)$ and that $(1-d)O_{S2\widehat{t}} \geq F(\theta^e)$. This is because all households with types θ less than θ^e will strictly prefer to purchase a small house in period $\widehat{t}$ at these prices. Thus, we must have that $O_{S2\widehat{t}} \geq F(\theta^e)/(1-d)$ in order for the housing market to clear. Now consider period $\widehat{t} - 1$. Suppose that $(1-d)O_{S2\widehat{t}-1} < F(\theta^e)$. Then, since $O_{S2\widehat{t}} \geq F(\theta^e)/(1-d)$, there must be new construction of small houses in community 2 in period $\widehat{t} - 1$. In that case, $P_{S2\widehat{t}-1} = C_S$, but since the price of small houses falls in period $\widehat{t}$, no households with types greater than θ^e will want to purchase a small house in community 2. Accordingly, we have that $(1-d)O_{S2\widehat{t}-1} + N_{S2\widehat{t}-1} \leq F(\theta^e)$. But then we have that

$$O_{S2\widehat{t}} = (1-d)O_{S2\widehat{t}-1} + N_{S2\widehat{t}-1} < F(\theta^e)/(1-d),$$

which is a contradiction. Thus, $O_{S2\widehat{t}-1} \geq F(\theta^e)/(1-d)$. This in turn implies that $P_{L1\widehat{t}-1} =$

C_L, $P_{S2\hat{t}-1} < C_S$, and that $N_{L1\hat{t}-1} = d$. Again, there can be no new construction of small houses, because all households of type greater than θ^e will want large houses. Continuing this line of argument, we conclude that for all $t = 1, ..., \hat{t}$, we must have that $O_{S2t} \geq F(\theta^e)/(1-d)$. But since $O_{S20} = F(\theta^e)$ and $N_{S20} = 0$, we have that

$$O_{S21} = (1-d)O_{S20} + N_{S20} = (1-d)F(\theta^e) < F(\theta^e)/(1-d)$$

which is a contradiction. We conclude therefore that it cannot be that $P_{L1\hat{t}} = C_L$, $P_{S2\hat{t}} < C_S$, and $N_{L1\hat{t}} = d$.

We have therefore established that in period $\hat{t}$, $P_{L1\hat{t}} < C_L$, $P_{S2\hat{t}} = C_S$, and $N_{S2\hat{t}} = d$. If $\hat{t} \geq 2$, consider period $\hat{t}-1$. Again, there are two possibilities: (i) $P_{L1\hat{t}-1} = C_L$, $P_{S2\hat{t}-1} < C_S$, and $N_{L1\hat{t}-1} = d$, and (ii) $P_{L1\hat{t}-1} < C_L$, $P_{S2\hat{t}-1} = C_S$, and $N_{S2\hat{t}-1} = d$. Using similar logic, we can again show that possibility (i) cannot arise.

Continuing on in this way, we conclude that for all $t = 1, ..., \hat{t}$, we have that $P_{L1\hat{t}} < C_L$, $P_{S2\hat{t}} = C_S$, and $N_{S2\hat{t}} = d$. Now consider period 0, the period the deviation becomes effective. We know that $O_0 = (1 - F(\theta^e), 0, 0, F(\theta^e))$, that $N_0 = (d, 0, 0, 0)$ and that $P_{L10} = C_L$. We also know that $P_{L11} \leq C_L$ and that $P_{S21} = C_S$. We now argue that $P_{S20} > C_S$. Suppose to the contrary that $P_{S20} \leq C_S$. Then, it must be the case that in period 0 all types with preferences less than θ^e strictly prefer small houses in community 2 implying that demand is at least equal to $F(\theta^e)$. Supply, however, is equal to $(1-d)F(\theta^e)$. ■

We can now complete the proof of the Proposition. Consider the payoff of a household of type 0 under the deviation. As the household with the lowest preference for large houses, this household can expect to remain in small houses in community 2 for as long as it remains in the area. Thus, given that for all $t \geq 1$, $P_{S2t} = C_S$ and $\lambda_{2t} = 0$, we have that

$$V_0(O^*, (1,1)) = y + B(g^e) - cg^e - P_{S20} + \delta[(1-d)C_S + \mu V^* + (1-\mu)\frac{y}{1-\delta}].$$

This household will favor imposing zoning if

$$(1-d)P_{S20} + \mu V_0(O^*, (1,1)) + (1-\mu)\frac{y}{1-\delta} - \left[(1-d)C_S + \mu V^* + (1-\mu)\frac{y}{1-\delta}\right] > 0.$$

This difference equals

$$(1-\mu-d)\left[P_{S20} - C_S\right],$$

which is positive given that $1 - \mu > d$. It follows that households of type 0 are in favor of imposing zoning.

Now consider households of type $\theta \in (0, \theta^e]$. As noted, the continuation payoff for these residents if zoning is not implemented is exactly the same as for a type 0 household. On the other hand, since a type θ household can always make the same choices as a type 0 household, it must be the case that $V_\theta(O^*,(1,1)) \geq V_0(O^*,(1,1))$. It therefore follows that

$$\begin{aligned} & (1-d)P_{S20} + \mu V_\theta(O^*,(1,1)) + (1-\mu)\frac{y}{1-\delta} - \left[(1-d)C_S + \mu V^* + (1-\mu)\frac{y}{1-\delta}\right] \\ \geq \; & (1-\mu-d)\left[P_{S20} - C_S\right] > 0. \end{aligned}$$

Thus, households of type $\theta \in (0,\theta^e]$ also favor imposing zoning. ■

10.1.5 Extension of Proposition 3 to three communities

Note first there is a sense in which Proposition 3 is trivially true when we have three communities. Take an efficient steady state in which, say, communities 2 and 3 are unzoned and the small houses are allocated in community 2 so that $O^* = (1-F(\theta^e),0,0,F(\theta^e),0,0)$ and $Z(O^*) = (1,0,0)$. Consider for ε small and positive the stock $O = (1-F(\theta^e),0,0,F(\theta^e) - \varepsilon,\varepsilon,0)$. Thus, compared with O^*, O features a small number of large houses in community 3. Define $\langle O_t(O)\rangle_{t=0}^{\infty}$ in the usual way and assume that $Z(O_t(O)) = Z(O^*)$ for all t. Then it cannot be the case that $\lim_{t\to\infty} O_t(O) = O^*$. This is because, by virtue of having a small fraction of large houses, community 3 is now a more attractive place to build new small houses than community 2. This means the fraction of small houses in community 3 will grow relative to that in community 2 and $\lim_{t\to\infty} O_t(O) = (1-F(\theta^e),0,0,0,0,F(\theta^e))$. It follows that O^* is not strongly locally stable in the sense defined earlier.

Nonetheless, from an efficiency perspective there is nothing troubling about this example. Whether in the long run all the small housing is located in community 2, community 3, or both, is immaterial. The difficulty is that the division of construction across the two unzoned communities is arbitrary and small differences between the two communities will force it in one or the other direction. To reflect this, we modify our definition of strong local stability. For a given equilibrium with endogenous zoning and any given zoning rules Z, let $\Phi(Z)$ be the set of housing stocks O that are steady states and have the property that $Z(O) = Z$. Then, we say that a steady state O^* is strongly locally stable if there exists $\varepsilon > 0$ such that for any initial stock O with the property that $\|O - O^*\| < \varepsilon$, the sequence of housing stocks $\langle O_t((1-d)O + N)\rangle_{t=0}^{\infty}$ converges to some steady state in $\Phi(Z(O^*))$ for any arbitrary vector of new construction N such that $N_{L1} + N_{S1} + N_{L2} + N_{S2} = d$. Moreover, the associated zoning rules $\langle Z(O_t((1-d)O+N))\rangle_{t=0}^{\infty}$ equal $Z(O^*)$ in all periods. This definition reduces to our earlier one in the case in which there is a unique steady state associated with a particular set of zoning rules.

We now demonstrate that Proposition 3 holds with this new more general definition of strong local stability. Suppose, to the contrary, that there exists a political equilibrium with endogenous zoning with an equilibrium steady state O^* which is both efficient and strongly locally stable. It must be the case that $Z(O^*)$ involves at least one community zoning and one community not zoning. Without loss of generality, assume that $Z_1(O^*) = 1$ and $Z_3(O^*) = 0$. If $Z_2(O^*) = 1$, then community 3 is a monopoly supplier of small homes and we can use the the logic from Proposition 3 to obtain a contradiction. Thus, assume that $Z_2(O^*) = 0$. This implies that $O^* = (1 - F(\theta^e), 0, 0, F(\theta^e) - x, 0, x)$ for some $x \in [0, F(\theta^e)]$.

Now consider for ε small and positive the stock

$$O(\varepsilon) = (1 - F(\theta^e), 0, \varepsilon, F(\theta^e) - x - \varepsilon, 0, x).$$

This differs from the steady state in that community 2 has a small fraction of large homes. We will show that for sufficiently small ε, $Z_2(O(\varepsilon)) \neq 0$ which will contradict the assumption O^* is strongly locally stable. To see this recall that if O^* is strongly locally stable it must be the case that for sufficiently small $\varepsilon > 0$ we have that for any arbitrary vector of new construction N such that $N_{L1} + N_{S1} + N_{L2} + N_{S2} = d$, $\langle O_t((1-d)O(\varepsilon) + N)\rangle_{t=0}^{\infty}$ converges to some steady state in $\Phi((1, 0, 0))$ and the associated zoning rules $\langle Z(O_t((1-d)O(\varepsilon) + N))\rangle_{t=0}^{\infty}$ equal $(1, 0, 0)$ in all periods. In particular, this must be true for $N = dO(\varepsilon)$, which implies that $Z(O(\varepsilon))$ must equal $(1, 0, 0)$.

Suppose first that the residents of community 2 follow the postulated equilibrium and do not impose zoning. As the efficient steady state is strongly locally stable, the future play of the equilibrium will have community 1 implementing zoning and communities 2 and 3 not. All new construction of small homes will occur in community 2 and the price of new small homes will be less than C_S in community 3. All new construction of large homes will occur in community 1 and the price of large homes in community 2 will be less than C_L. Let λ_{2t}^0 be the fraction of large homes (post-construction) in community 2 after $t = 0,....,\infty$ periods and let P_{L2t}^0 be the price of such houses.

Now suppose that the residents of community 2 were to deviate from the postulated equilibrium behavior by imposing zoning. By strong local stability, in all subsequent periods, households anticipate that zoning rules will return to steady state levels. In the period of the deviation, new construction of small homes will occur in community 3. However, the price of small homes in community 2 must be higher than those in community 3 because of the beneficial fiscal externality created by the presence of large homes. Let P_{S20}^1 be the price of small homes in community 2 in the period of the deviation. The value of large homes in community 2 will also be higher than on the equilibrium path as will the fraction of large homes. Let P^1_{L20} be the price of large homes in community 2 and λ^1_{20} the fraction. Following the period of deviation, there will be a lower fraction of small homes in community 2 which will increase the price of large homes relative to the equilibrium. Let λ^1_{2t} denote the fraction (post-construction) after t periods and let P^1_{L2t} denote the price. The price of small homes in community 2 will return to C_S and the price of small homes in community 3 will be less than the construction cost.

Now consider the incentives to deviate for the types who own small homes in community 2. The payoff on the equilibrium path for a type θ who owns a small house in community 2 and will continue to live in a small house is

$$(1-d)C_S + \mu V^0_0 + (1-\mu)\frac{y}{1-\delta}.$$

The continuation value V^0_0 is the first element of the sequence $\langle V^0_t \rangle^\infty_{t=0}$ defined inductively by

$$\begin{aligned} V^0_t &= B(g^*(\frac{cC_S}{\lambda^0_{2t}P^0_{L2t} + (1-\lambda^0_{2t})C_S})) - \frac{cC_S}{\lambda^0_{2t}P^0_{L2t} + (1-\lambda^0_{2t})C_S} g^*(\frac{cC_S}{\lambda^0_{2t}P^0_{L2t} + (1-\lambda^0_{2t})C_S}) - C_S \\ &\quad + \delta[(1-d)C_S + \mu V^0_{t+1} + (1-\mu)\frac{y}{1-\delta}] \end{aligned}$$

with end point condition

$$V^0_\infty = \frac{B(g^*(c)) - cg^*(c) - C_S + \delta[(1-d)C_S + (1-\mu)\frac{y}{1-\delta}]}{1-\delta\mu}.$$

The payoff under the deviation for a type θ who owns a small house in community 2 and will continue to live in a small house is

$$(1-d)P^1_{S20} + \mu V^1_0 + (1-\mu)\frac{y}{1-\delta},$$

where

$$\begin{aligned} V^1_0 &= B(g^*(\frac{cP^1_{S20}}{\lambda^1_{20}P^1_{L20} + (1-\lambda^1_{20})P^1_{S20}})) - \frac{cP^1_{S20}}{\lambda^1_{20}P^1_{L20} + (1-\lambda^1_{20})P^1_{S20}} g^*(\frac{cP^1_{S20}}{\lambda^1_{20}P^1_{L20} + (1-\lambda^1_{20})P^1_{S20}}) - P^1_{S20} \\ &\quad + \delta[(1-d)C_S + \mu V^1_1 + (1-\mu)\frac{y}{1-\delta}]. \end{aligned}$$

The continuation value V^1_1 is the first element of the sequence $\langle V^1_t \rangle^\infty_{t=1}$ defined inductively by

$$\begin{aligned} V_t^1 &= B(g^*(\frac{cC_S}{\lambda_{2t}^1 P_{L2t}^1 + (1-\lambda_{2t}^1)C_S})) - \frac{cC_S}{\lambda_{2t}^1 P_{L2t}^1 + (1-\lambda_{2t}^1)C_S} g^*(\frac{cC_S}{\lambda_{2t}^1 P_{L2t}^1 + (1-\lambda_{2t}^1)C_S}) - C_S \\ &+\delta[(1-d)C_S + \mu V_{t+1}^1 + (1-\mu)\frac{y}{1-\delta}] \end{aligned}$$

with end point condition

$$V_\infty^1 = \frac{B(g^*(c)) - cg^*(c) - C_S + \delta[(1-d)C_S + (1-\mu)\frac{y}{1-\delta}]}{1-\delta\mu}.$$

The gain from deviating is

$$\Delta = (1-d)[P_{S20}^1 - C_S] + \mu\left[V_0^1 - V_0^0\right]$$

But we have that

$$V_0^1 - V_0^0 = S_0^1 - P_{S20}^1 + \delta\mu V_1^1 - \left[S_0^0 - C_S + \delta\mu V_1^0\right]$$

where S_t^0 and S_t^1 denote public service surplus on the equilibrium path and with the deviation. Thus, we have that

$$\Delta = (1-\mu-d)[P_{S20}^1 - C_S] + \mu\left[S_0^1 - S_0^0\right] + \delta\mu^2\left[V_1^1 - V_1^0\right]. \tag{31}$$

We now claim that

$$\mu\left[S_0^1 - S_0^0\right] \geq -\mu c g^*(\frac{cC_S}{\lambda_{20}^0 P_{L20}^0 + (1-\lambda_{20}^0)C_S})\left(\frac{[P_{S20}^1 - C_S]\,\lambda_{20}^1 C_L}{C_S^2}\right). \tag{32}$$

To prove this, note that

$$g^*(\frac{cP_{S20}^1}{\lambda_{20}^1 P_{L20}^1 + (1-\lambda_{20}^1)P_{S20}^1}) = \arg\max_g\left\{B(g) - \frac{cP_{S20}^1}{\lambda_{20}^1 P_{L20}^1 + (1-\lambda_{20}^1)P_{S20}^1} g\right\}$$

and so

$$S_0^1 \geq B(g^*(\frac{cC_S}{\lambda_{20}^0 P_{L20}^0 + (1-\lambda_{20}^0)C_S})) - \frac{cP_{S20}^1}{\lambda_{20}^1 P_{L20}^1 + (1-\lambda_{20}^1)P_{S20}^1} g^*(\frac{cC_S}{\lambda_{20}^0 P_{L20}^0 + (1-\lambda_{20}^0)C_S}).$$

It follows that

$$\mu\left[S_0^1 - S_0^0\right] \geq \mu c g^*(\frac{cC_S}{\lambda_{20}^0 P_{L20}^0 + (1-\lambda_{20}^0)C_S})\left[\frac{cC_S}{\lambda_{20}^0 P_{L20}^0 + (1-\lambda_{20}^0)C_S} - \frac{cP_{S20}^1}{\lambda_{20}^1 P_{L20}^1 + (1-\lambda_{20}^1)P_{S20}^1}\right]$$

Moreover, since $P^1_{L20} \geq P^0_{L20}$, $P^1_{L20} \geq C_S$ and $\lambda^1_{20} > \lambda^0_{20}$, we have that

$$\begin{aligned} &\mu c g^*(\cdot)\left[\frac{C_S}{\lambda^0_{20}P^0_{L20}+(1-\lambda^0_{20})C_S}-\frac{P^1_{S20}}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})P^1_{S20}}\right] \\ \geq\ &\mu c g^*(\cdot)\left[\frac{C_S}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})C_S}-\frac{P^1_{S20}}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})P^1_{S20}}\right] \end{aligned}$$

In addition,

$$\mu c g^*(\cdot)\left[\frac{C_S}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})C_S}-\frac{P^1_{S20}}{\lambda^1_{20}P^1_{L20}+(1-\lambda^1_{20})P^1_{S20}}\right] \geq -\mu c g^*(\cdot)\left[\frac{\left(P^1_{S20}-C_S\right)\lambda^1_{20}P^1_{L20}}{C_S^2}\right].$$

Combining (31) and (32), we have

$$\Delta \geq (1-\mu-d)[P^1_{S20}-C_S] - \mu c g^*(\cdot)\left[\frac{\left(P^1_{S20}-C_S\right)\lambda^1_{20}P^1_{L20}}{C_S^2}\right] + \delta\mu^2\left[V^1_1 - V^0_1\right]. \tag{33}$$

We also claim that $V^1_1 \geq V^0_1$. For this, it is enough to show that for all $t = 1, ..., \infty$, $S^1_t \geq S^0_t$. We have that

$$S^1_t \geq B\left(g^*\left(\frac{cC_S}{\lambda^0_{2t}P^0_{L2t}+(1-\lambda^0_{2t})C_S}\right)\right) - \frac{cC_S}{\lambda^1_{2t}P^1_{L2t}+(1-\lambda^1_{2t})C_S}g^*\left(\frac{cC_S}{\lambda^0_{2t}P^0_{L2t}+(1-\lambda^0_{2t})C_S}\right).$$

Thus,

$$S^1_t - S^0_t \geq \left[\frac{1}{\lambda^0_{2t}P^0_{L2t}+(1-\lambda^0_{2t})C_S}-\frac{1}{\lambda^1_{2t}P^1_{L2t}+(1-\lambda^1_{2t})C_S}\right] cC_S g^*\left(\frac{cC_S}{\lambda^0_{2t}P^0_{L2t}+(1-\lambda^0_{2t})C_S}\right)$$

But we know that $\lambda^1_{2t} \geq \lambda^0_{2t}$, $P^1_{L2t} \geq P^0_{L2t}$ and $P^0_{L2t} \geq C_S$. Thus, we have that

$$\frac{1}{\lambda^0_{2t}P^0_{L2t}+(1-\lambda^0_{2t})C_S} \geq \frac{1}{\lambda^1_{2t}P^1_{L2t}+(1-\lambda^1_{2t})C_S},$$

which implies the result.

It now follows from (33) that

$$\Delta \geq [P^1_{S20}-C_S]\left((1-\mu-d) - \mu c g^*(\cdot)\left[\frac{\lambda^1_{20}P^1_{L20}}{C_S^2}\right]\right)$$

Since $\lambda^1_{20} \leq \varepsilon / \left(F(\theta^e) - x\right)$, this must be positive for sufficiently small ε given that $1-\mu > d$. ■

10.2 Computing equilibrium

10.2.1 Equilibrium with no zoning

To compute the equilibrium with no zoning, we rely on a "guess and verify" approach. For any initial stock, we first conjecture how the housing stock will evolve and compute the limiting housing prices. Working backwards, we then use the market equilibrium conditions developed in Section 4.2 to construct the implied sequence of housing prices. Knowing these and the evolution of the housing stock, we then recover taxes and public service levels. The exact details of the procedure depend upon the particular initial stock we are working with. To illustrate, we explain the procedure for the two scenarios illustrated in Figures 1 and 2.

Scenario 1 In this scenario, the two communities begin with the same housing stocks and, in our conjectured equilibrium, they remain symmetric. The evolution of the housing stock is as follows. Starting from any symmetric allocation, housing stocks will jump to the steady state, if such a jump is feasible given the constraint that new construction must sum to d. If this is not feasible, there will be construction of large homes if the share of large homes is below the steady state level and construction of small homes otherwise. Given this law of motion, the housing stock arrives at the steady state in a finite number of periods. The exact number depends upon the fraction of large houses in the initial state. The housing prices in the "jump" period in which the stock first reaches the steady state will equal their respective construction costs.

Given this knowlege of how the housing stock will evolve and the limiting prices, the prices in the preceding periods are constructed by backwards induction. Given the symmetry of the allocation, the prices will be equal across comunities and hence we just have to solve for two sequences $\langle P_{St}\rangle_{t=1}^{T}$ and $\langle P_{Lt}\rangle_{t=1}^{T}$ where T denotes the period in which the stock arrives at the steady state. For each period, from our knowledge of the evolution of the housing stock, we can compute θ_c from equation (15). Moreover, one of the current prices is known since P_{St} is equal to C_S if small homes are being built, and P_{Lt} is equal to C_L otherwise. Equation (14) gives us an equation in the unknown price because public service levels and tax prices can be expressed as functions of prices (and the housing stock) using (7), (8), and (9). Using this equation, we can solve for the sequence of unknown prices. We do this, using a backward shooting algorithm. This requires solving iteratively (T times) one equation in one unknown. Once we have done this, we can recover the public service levels and taxes from equations (7), (8), and (9).

Scenario 2 In this scenario, one community (community 1) has a larger fraction of large homes than the other, but with both still less than the steady state. In our conjectured equilibrium, at first all new construction is in the form of large homes and occurs in community 1. After a finite number of periods, small home construction begins and also occurs in community 1. Eventually, community 2 will become so small that one period's new construction of large homes will be sufficient to equate the fraction of large homes in the two communities. At this point, the housing stock jumps to the steady state.

The precise timing of this evolution is constructed using a backwards shooting algorithm. First, the period in which the stock jumps to the steady state is conjectured. Expression (14) is used to track backwards the evolution of the stock under the assumption that both types of homes are built in community 1 and hence prices in that community are equal to construction costs. Going backwards, when the housing stock arrives to a point at which the size of community 1 and the number of large homes are such that it could be reached from the initial state in a finite number of periods by building only large homes, the construction of small homes ceases. From then on, again going backwards, all construction is of large homes, and hence the evolution of the stock is known. Equation (14) is used to construct the prices for small homes in community 1 during these periods.

This procedure reveals the evolution of the stock and the price path in community 1. From this, we can compute the public service levels and taxes for community 1. In community 2, there is no new construction until the steady state is reached, so the prices of both home types need to be determined. We employ equation (12) for both home types. Given period $t+1$ home prices in community 2, these two equalities provide two equations in the two unknown period t prices because period t public service levels and tax prices can be expressed as functions of period t home prices in community 2. Hence community 2's prices can be obtained by solving iteratively these two equations in two unknowns. Again, once we have community 2's prices, we also have the public service levels and taxes for community 2 from equations (7), (8), and (9).

10.2.2 Equilibrium with exogenous zoning

A different procedure is required to compute the equilibrium with exogenous zoning. This is because guessing the precise evolution of the housing stock is too difficult. While we know that new construction will involve only large homes in community 1 and small homes in community 2, we do not know the precise mix.

The first step in computing the equilibrium is to find the limiting housing prices. We know that the zoned community (community 1) converges asymptotically to one with large homes only and size $1-\theta^e$, and the unzoned community (community 2) to one with small homes only and size θ^e. The limiting prices of large homes in community 1 and of small homes in community 2 are just the construction costs. Since we know θ^c (it equals θ^e), the prices of small homes in community 1 and large homes in community 2 can be obtained from (14).

The second step is to compute the evolution of the housing stock. For this problem, a backwards shooting algorithm cannot be implemented as the stock in period $t+1$ does not necessarily pin down the stock in period t. Instead, we use a variant of a forward shooting algorithm, as described below:

- We set the number of periods for convergence to the steady state to occur. Given the slow decay of the housing stock, a large number of periods are required. The number we use is 400. With this number of periods, the difference between the community sizes in the final period and their steady state values is less than 10^{-4}.

- Since there are only two types of homes that are built, the evolution of the stock can be described by the sequence of new large homes built in community 1 $\langle N_{L1t}\rangle_{t=1}^{400}$. The associated small home construction is given by $N_{S2t} = d - N_{L2t}$.

- We assume first that all N_{L1t} are interior: that is $0 < N_{L1t} < d$, for all t. This implies there is always construction of both home types. This assumption ties down the prices of large homes in community 1 and of small homes in community 2. Given the limit prices, the remaining prices can be computed using equation (14) by backwards induction starting in period 400 in which, by assumption, the stock has converged to the steady state. We then note that condition (12) does not hold for any such sequence – it holds only for the equilibrium one. This suggests the following iterative updating.

  1. Make the initial guess for $\langle N_{L1t}\rangle_{t=1}^{400}$ and compute the associated housing prices. (Our guess is that each element in the sequence is equal to its long run level, $(1-\theta^e)d$).
  2. Start updating the guess in period $t = 1$. Taking period 2 prices from step 1, solve the following three equations for the three unknowns $(N_{L1t}, P_{S1t}, P_{L2t})$: equation (14) written for both communities and expression (12) written for large (or small) homes.
  3. Update the guess for N_{L1t} and (P_{S1t}, P_{L2t}).
  4. Go to the next period and repeat steps 2 and 3. Keep going until the period in which by assumption the steady state is reached (i.e., period 400). The new guess for the entire path for $\langle N_{L1t}\rangle_{t=1}^{400}$ and prices is now constructed.

5. Keep repeating the steps of the above procedure until the maximum difference between the elements in the "old" and "new" construction sequences is less than some specified tolerance (10^{-5} in our case).

- The algorithm above converges, but violates the assumption that N_{L1t} is interior in the first four periods. The algorithm assigns it a value of d, which suggests that in equilibrium in these periods there is no construction of small homes in community 2. Hence we modify the algorithm – in these periods, instead of searching for $(N_{L1t}, P_{S1t}, P_{L2t})$ we postulate that N_{L1t} is equal to d and use the expressions (14) written for both communities and expression (12) to compute the triplet of prices $(P_{S1t}, P_{S2t}, P_{L2t})$. With this modification the algorithm converges.

The iterative procedure just described for computing the evolution of the housing stock simultaneously determines the sequence of housing prices. It only remains to recover the public service levels and taxes for the two communities which we do using equations (7), (8), and (9).

10.2.3 Equilibrium with endogenous zoning

In our always-zone equilibrium, both communities impose zoning except when a community is empty. New construction occurs in the community which has the largest share of large homes. This is determined by the initial stock, since the community with the highest initial stock of large homes will remain the community with the largest fraction for eternity. By relabeling as necessary, we can call the community with the highest initial share community 1. If the initial stock is such that the two communities have an equal fraction of large homes, then all new construction occurs in community 1.

In this equilibrium, the evolution of the housing stock is straightforward: all new construction is in large homes and it occurs in community 1. This community grows in size and the fraction of its homes that are large converges to one. Community 2 converges in size to zero and the fraction of its homes that are large remains constant.

With the evolution of the housing stock determined, the next step is to compute the associated housing prices. The first point to note is that since the distribution of preference types has full support on $[0, 1]$ there is a positive measure of households who are almost indifferent between large and small homes. Since the fraction of large homes converges to one in the limit, the limiting price of all homes will equal the replacement cost of large homes C_L. Knowing this, and the evolution of the housing stock, enables us to compute home prices in both communities. Given the asymptotic convergence to the limit allocation, a large number of periods is required. As in the case with exogenous zoning, we set this number equal to 400.

In community 1, the price of large homes is always C_L since this is where new construction takes place. From our knowledge of the evolution of the housing stock, we can compute θ_c from equation (15). Using this together with (7), (8), and (9), we can solve for the sequence of small home prices in community 1 $\langle P_{S1t}\rangle_{t=1}^{400}$ from equation (14). As in the no zoning case, we do this using a backward shooting algorithm. This requires solving iteratively (400 times) one equation in one unknown. Once we have done this, we can recover the public service levels and taxes for community 1 from equations (7), (8), and (9).

In community 2, there is no new construction, so the prices of both home types need to be determined. Thus, we have to solve for both $\langle P_{L2t}\rangle_{t=1}^{400}$ and $\langle P_{S2t}\rangle_{t=1}^{400}$. We employ equation (12) for both home types. Given period $t+1$ home prices in community 2, these two equalities provide two equations in the two unknown period t prices because period t public service levels and tax prices can be expressed as functions of period t home prices in community 2. Hence community 2's prices can be obtained by solving iteratively these two equations in two unknowns. Again, once we have community 2's prices, we also have the public service levels and taxes for community 2 from equations (7), (8), and (9).

The main additional complication associated with computing this equilibrium is checking that the majority of residents of each community support zoning. To do this, the first step is to calculate each household type's equilibrium value function. With information about the entire sequence of housing prices, public service levels and taxes, we can compute households' equilibrium value functions from equation (4). Again, backward induction is employed. In the limiting allocation, the value functions for any household type θ with a given type of home in either community is straightforward to compute. Going backward, for each θ we first compute its (within) period housing decision and based on that decision we compute the (beginning of the period) value from owning a home of given type in a given community.

The second step is to checking that for all housing stocks equation (25) is satisfied for a majority of the residents living in each community given that the other community is choosing zoning and given that both communities will play according to the equilibrium in the future. Since it is infeasible to check the condition for every conceivable housing stock, it is necessary to limit the housing stocks we consider. Given that the housing stock $(O_{L1}, O_{S1}, O_{L2}, O_{S2})$ must have aggregate size 1, we can represent any stock as a triplet: the share of large homes in community 1, the share of large homes in community 2, and the size of community 1. We consider a grid of housing stocks in the three dimensional simplex with 13824 (24^3) points. The grid is uniform on $[0,1]^3$, except any 0 is substituted by 0.01 and any 1 is substituted by 0.99.

In order to check equation (25), we need to understand what would happen if one community deviated from the equilibrium by removing zoning. The key task is to solve for prices and new construction in the period of the deviation. This new construction determines the housing stock in the following period. From then on we know what will happen because play follows the equilibrium and the steps just described allow us to solve for the equilibrium path for any initial state. The housing market equilibrium in the period of the deviation is computed by searching for an allocation such that equations (12) and (14) hold when evaluated at the future prices implied by the housing stock generated by this construction allocation.

With information about the entire sequence of housing prices, public service levels and taxes following a deviation, we can compute households' payoffs from the deviation and check whether equation (25) is indeed satisfied for a majority of the residents living in each community. In order to know which households are voting in which district, the equilibrium needs to specify how household types are allocated across the two communities. This is done as follows. Suppose the stock is $(O_{L1}, O_{S1}, O_{L2}, O_{S2})$ and recall that households types are uniformly distributed on $[0, 1]$. Then, in equilibrium all households with types θ less than $O_{S1} + O_{S2}$ are living in a small home and all those with θ above $O_{S1} + O_{S2}$ in a large home. Our equilibrium assumes that for any household type θ less than $O_{S1} + O_{S2}$, a fraction $O_{S1}/\left(O_{S1} + O_{S2}\right)$ of this type live in community 1 and a fraction $O_{S2}/\left(O_{S1} + O_{S2}\right)$ live in community 2. Similarly, for any household type θ exceeding $O_{S1} + O_{S2}$, a fraction $O_{L1}/\left(O_{L1} + O_{L2}\right)$ of this type live in community 1 and a fraction $O_{L2}/\left(O_{L1} + O_{L2}\right)$ live in community 2. As an example, suppose that $(O_{L1}, O_{S1}, O_{L2}, O_{S2}) = (1/3, 1/6, 1/6, 1/3)$. Then $1/3$ of each type θ less than $1/2$ live in community 1 and $2/3$ live in community 2. Similarly, $2/3$ of each type θ greater than $1/2$ live in community 1 and $1/3$ live in community 2. To aggregate preferences in a given community we need to integrate over the distribution of citizens by their θ type and home ownership. We discretize the distribution of θ using an equi-spaced 51 point grid on [0,1].