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Evaluating the Social Optimality of Durable Public Good Provision using the Housing Price Response to Public Investment*

Abstract

Recent empirical work in public finance uses the housing price response to public investment to assess the efficiency of local durable public good provision. This paper explores the theoretical justification for this technique. It points out that the logic justifying the technique for evaluating non-durable public good provision does not translate to the durable case. A model in which investment is determined by the interaction between a budget-maximizing bureaucrat and a community's residents is used to demonstrate that the technique can falsely predict under-provision, falsely predict over-provision, or perform without error.

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1 Introduction

A sizable fraction of government spending is devoted to investment in durable public goods. Such investment is undertaken by all levels of government - federal, state, and local. The goods in question include physical infrastructure (roads, bridges, airports, etc), basic research, defense equipment, environmental clean-ups, parks, and schools. A basic question of interest to economists and policy-makers is how the levels of durable public good provision emerging from the political process compare with socially optimal levels. This question arises in many different policy areas. For example, there seems broad agreement that government substantially underinvests in physical infrastructure and basic research. There is much less agreement concerning defense, environmental, and educational investments, with conservatives and liberals often coming down on opposing sides of the issue. Given the importance of the question, it would be helpful if economic analysis provided convincing ways of answering it.

There is a long tradition in public finance of using housing prices to assess the social optimality of local non-durable public good provision (see, for example, Brueckner 1979, 1982, Lind 1973, and Wildasin 1979). The underlying idea is that the demand of potential residents to live in a community will be influenced by the local public goods it provides and the taxes it levies to finance them (Oates 1969, Tiebout 1956). Accordingly, the net surplus generated by local public good provision will be reflected in housing prices.[1] In a well-known and elegant theoretical formulation of the idea, Brueckner develops a model in which if housing prices rise following a small, permanent increase in local non-durable public good provision, then it can be inferred that the good is under-provided. Conversely, if housing prices fall, the good is over-provided (Brueckner 1979, 1982). This model has been used as the basis for a number of empirical studies of the optimality of local public good provision (see, for example, Barrow and Rouse 2004, Brueckner 1982, and Lang and Jian 2004).

Can housing prices be used to assess the social optimality of local durable public good provision? In an ambitious and creative paper, Cellini, Ferreira, and Rothstein (2010) employ the approach to detect whether local school districts are over or under-providing public school facilities. Using a static version of Brueckner's model, they argue that if housing prices in a district rise

[1] A vast literature investigates the relationship between housing prices and local public good provision empirically, with particular focus on schooling. See Nguyen-Hoang and Yinger (2011) and Ross and Yinger (1999) for useful surveys.

following an investment in public school facilities, then such facilities are under-provided. Conversely, if housing prices fall, facilities are over-provided. To estimate what house prices would be in the counter-factual situation in which an observed investment is not undertaken, Cellini, Ferreira, and Rothstein exploit the fact that investments must be approved by residents in a referendum. Drawing on the regression discontinuity literature, they then compare housing prices in school districts in which referenda have just passed with those in which they have just failed. If prices are higher in the just passing districts, they argue that school facilities are under-provided. This is indeed what they find for California school districts.

The intuitive appeal of this approach not withstanding, there are important conceptual differences between an investment in a durable public good and a permanent increase in a non-durable public good. First, because of depreciation, the benefits from investment in the durable public good will not be permanent. Rather, they will diminish over time. Second, again because of depreciation, whether or not the investment in question is undertaken, future investments will be made by the community. Moreover, the nature of these investments will depend on the stock of the public good and hence on the fate of the investment in question. This creates a linkage between the current investment and the future investment path in the community. These differences raise the question of whether the logic that underlies the non-durable case can be applied to justify using the housing price response to public investment to evaluate durable public good provision. The purpose of this paper is to investigate this important question.

The paper begins by developing a simple model to study the issue. This model is designed to capture the recurring nature of investment in durable public goods and the linkages between decision-making periods that durability creates. The model is a partial equilibrium model of a single community whose government provides a durable public good. There is a pool of households who, for exogenous reasons, are potential residents of the community. Households move in and out of this pool, creating an active housing market in each period. Investment in the public good is financed by a tax on the residents. The path of investment follows an exogenous stochastic process, reflecting the machinations of an unmodelled political process. The supply of houses in the community is perfectly inelastic, implying that the future surplus a resident is expected to receive from public good provision is fully capitalized into housing prices.[2]

[2] This paper does not contribute to the important debate in the literature about the extent to which the property capitalization effects offer reliable estimates of households' willingness to pay for changes in local public goods and

This model is used to investigate whether the housing price response to investment reveals the efficiency of durable public good provision in the community. In particular, it investigates the accuracy of the *housing price test* which asserts that i) a non-negative housing price response to an investment implies that the public good level *without the investment* is too low, and ii) a non-positive housing price response implies that the public good level *with the investment* is too high. It shows that the housing price test works if the socially optimal level of the public good maximizes the surplus residents are expected to receive from provision *in equilibrium*; that is, when future investment follows the stochastic process that governs the evolution of the public good in the community. The paper points out that there is no reason to expect this to be the case. The argument is basically an application of the *theory of the second best* (Lipsey and Lancaster 1956). The socially optimal level of the public good is derived under the assumption that all future investments will be socially optimal. This will not generally be the case in equilibrium; if it were, the question of whether provision is optimal would be moot. Accordingly, the second best impacts of the current investment on the future investment path must be taken into account in the surplus maximization problem. This implies that the public good level that maximizes surplus in equilibrium could be much different from the socially optimal level.

When the socially optimal level of the public good does not maximize the residents' equilibrium public good surplus, the housing price test can fail. It can *falsely predict under-provision* when the housing price response to an investment is non-negative but the public good level without the investment is already too high or it can *falsely predict over-provision* when the housing price response is non-positive but the public good level with the investment is too low. At a general level, when and how the housing price test will fail is difficult to assess because it will depend on the details of the community's investment path. To shed more light on what can happen, the paper endogenizes the community's investment path with a political economy model. This model, an extended version of that developed by Barseghyan and Coate (2014), assumes that the investment path is generated by the interaction between the community's residents and a budget-maximizing bureaucrat who cares about the level of the public good but not its cost. Budget-maximization is a common assumption in the political economy literature and underlies Romer and Rosenthal's famous *agenda setter model*, the leading alternative to the median voter

amenities (see, for example, Klaiber and Smith 2013 and Kuminoff and Pope 2014). Instead, the paper is interested in the narrower question of whether housing price changes reveal the social optimality of local public good provision in an environment in which capitalization of future public good surplus occurs.

model of local government spending (Romer and Rosenthal 1978, 1979).[3] With this model, three examples are developed: one in which the housing price test falsely predicts under-provision, one in which it falsely predicts over-provision, and one in which it performs without error. These examples illustrate the unpredictable performance of the test.

On a more positive note, the paper points out that a justification for the test is available if the usual economic assumption that agents have rational expectations concerning the future path of policies is abandoned. Specifically, the test is shown to be accurate if households have adaptive expectations, believing that whatever level of public good they observe in the community at the beginning of a period will be maintained indefinitely. Thus, they observe the current quantity and quality of school facilities, say, and just assume they are at steady state levels. This is a form of myopia that is perhaps not too implausible, particularly for new residents moving into a community. This assumption means that residents perceive a successful investment as permanently increasing provision and this brings us back into the world studied by Brueckner.

Beyond providing a framework to analyze the theoretical question at hand, the model developed here makes a broader contribution. In particular, it provides a simple dynamic model of a housing market in which the market is active in each period and agents are rational and forward-looking. The model highlights the relationship between housing prices and fiscal variables, illustrating the phenomenon of capitalization. In contrast to standard treatments of capitalization in which values of policy variables are frozen through time, the model shows that it is both the current and future values of policy variables that are capitalized into housing prices. As we point out below, the significance of such dynamic considerations is being increasingly recognized in empirical studies. Moreover, by endogenizing the policy choices that would arise with a budget-maximizing bureaucrat, the model takes a first step in the direction of exploring the implications of different political processes for the dynamics of investment and housing prices. More generally, the paper fits in with a growing literature that studies issues in housing markets using dynamic models with rational, forward-looking households (see, for example, Bayer, McMillan, Murphy, and Timmins 2011, Bishop and Murphy 2011), particularly those papers that endogenize policy choices with political economy models (Barseghyan and Coate 2015, Epple, Romano, and Sieg 2012, and Ortalo-

[3] The budget maximizing assumption was first proposed by Niskanen (1971). For analysis of the relative performance of the median voter and agenda setter models see Romer and Rosenthal (1982), Romer, Rosenthal, and Munley (1992), and, in the specific context of school infrastructure investment, Balsdon, Brunner, and Rueben (2003).

Magne and Prat 2011).

The paper also contributes to a growing literature on durable public goods. While the vast majority of the public good literature has focused on the provision of non-durable goods, such as firework displays and police protection, in practice many important public goods are durable. Durability not only complicates the conditions for efficient provision but also makes understanding political provision considerably more challenging. This is because today's political choices have implications for future choices, creating a dynamic linkage across policy-making periods. The practical importance of durable public goods and the theoretical challenges they pose is leading to increasing interest in their provision. A number of recent papers have studied the provision of such goods under varying political institutions (see, for example, Battaglini and Coate 2007, Battaglini, Nunnari, and Palfrey 2012, 2015, Barseghyan and Coate 2014, and LeBlanc, Snyder, and Tripathi 2000). This paper shows that durability also has important implications for the evaluation of public good provision.

The organization of the remainder of the paper is as follows. Section 2 introduces the model. Section 3 characterizes socially optimal public good provision, establishing the normative benchmark departures from which the housing price test is supposed to detect. Section 4 explains why the logic underlying using the housing price response to a permanent increase in provision to evaluate non-durable public goods does not carry over to the durable case. Section 5 assumes that the community's investment path is generated by the interaction between residents and a budget-maximizing bureaucrat and provides the examples which show the different ways in which the housing price test may (mis)perform. Section 6 points out the adaptive expectations justification for the housing price test and Section 7 concludes.

2 The model

Consider a community such as a municipality or school district. This community can be thought of as one of a number in a particular geographic area. The time horizon is infinite and periods are discrete. There is a pool of potential residents of the community of size 1. These can be thought of as households who for exogenous reasons (employment opportunities, family ties, etc) need to live in the geographic area in which the community is situated. Potential residents are characterized by their desire to live in the community (as opposed to an alternative community in the area) which is measured by the preference parameter θ. This desire, for example, may be determined by a household's idiosyncratic reaction to the community's natural amenities. The preference parameter takes on values between 0 and $\overline{\theta}$, and the fraction of potential residents with preference below $\theta \in [0, \overline{\theta}]$ is $\theta/\overline{\theta}$. Reflecting the fact that households' circumstances change over time, in each period new households join the pool of potential residents and old ones leave. The probability that a household currently a potential resident will be one in the subsequent period is μ. Thus, in each period, a fraction $1 - \mu$ of households leave the pool and are replaced by an equal number of new ones.

The only way to live in the community is to own a house. There are a fixed number of houses sufficient to accommodate a population of size H where H is less than the size of the pool of potential residents (i.e., $H < 1$). These houses are infinitely durable.

The community provides a durable public good which depreciates at rate $\delta \in (0, 1)$. The level of the public good evolves according to a stochastic process. This process is such that if the current level of the public good is g, next period's level will be $(1 - \delta)g + I(g)$ with probability $\pi(g)$ and $(1 - \delta)g$ with probability $1 - \pi(g)$. The value $I(g)$ (which is non-negative) is to be interpreted as the investment in the public good proposed by the community's political process and $\pi(g)$ the probability that this investment is implemented. For now, the *investment path* (i.e., $I(g)$ and $\pi(g)$) is assumed to be exogenous, emerging from some unmodelled political process. Later in the paper, the path will be endogenized via an explicit model of the community's political process. Investment in the public good costs c per unit and is financed by a tax on those choosing to reside in the community. The community pays for the investment when it is complete and thus taxes to finance the investment are levied in the next period.[4]

When living in the community, households have preferences defined over the public good and consumption. A household with preference parameter θ and consumption x obtains a period payoff of $\theta + x + B(g)$ if living in the community with a public good level g. The benefit function $B(g)$ is increasing, smooth, strictly concave, and satisfies $B(0) = 0$. When not living in the community, a household's per period payoff is $\underline{u}$.[5] Households discount future payoffs at rate β and can borrow and save at rate $1/\beta - 1$. This assumption means that households are indifferent to the intertemporal allocation of their consumption. Each household in the pool receives an exogenous

[4] As will become clear below, the predictions of the model concerning the impact of an investment on the price of housing would not be changed if the cost of investment was financed by a bond issue rather than a tax.

[5] Note that $\underline{u}$ is both the per period payoff of living in one of the other communities in the geographic area if a household is in the pool and the payoff from living outside the area when a household leaves the pool.

income stream the present value of which is sufficient to pay taxes and purchase housing in the community.[6]

There is a competitive housing market which opens at the beginning of each period. Demand comes from new households moving into the community and supply comes from owners leaving the community. The price of houses is denoted P.

The timing of the model is as follows. Each period, the community starts with a public good level g and a tax obligation T (which may be zero). The tax obligation is to finance any investment implemented in the prior period. At the beginning of the period, those households in the pool learn whether they will be remaining and new households join. Those in the pool then decide whether to live in the community. The housing market opens and the equilibrium housing price $P(g,T)$ is determined. The community levies taxes on residents sufficient to meet its tax obligation and residents obtain their payoffs from living in the community. Next period's public good level and tax obligation (g', T') is then determined. With probability $\pi(g)$ it equals $((1-\delta)g + I(g), cI(g))$ and with probability $1-\pi(g)$ it equals $((1-\delta)g, 0)$.

2.1 Housing market equilibrium

We now explain how the housing market equilibriates taking as given the community's investment path.

Decisions of households At the beginning of any period, households fall into two groups: those who resided in the community in the previous period and those who did not, but could in the current period. Households in the first group own homes. The second group do not. Households in the first group who leave the pool sell their houses and obtain a continuation payoff of

$$P(g,T) + \frac{u}{1-\beta}. \tag{1}$$

The remaining households in the first group and all those in the second must decide whether to live in the community. Formally, they make a location decision $l \in \{0,1\}$, where $l = 1$ means that they live in the community. This decision will depend on their preference parameter θ, current and future housing prices, and public goods and taxes. Since selling a house and moving is costless, there is no loss of generality in assuming that all households sell their property at the beginning of

[6] The assumption that utility is linear in consumption means that there are no income effects, so it is not necessary to be specific about the income distribution.

any period.[7] This makes each household's location decision independent of its property ownership state. It also means that the only future consequences of the current location choice is through the selling price of housing in the next period.

To make this more precise, let $V_\theta(g,T)$ denote the expected payoff of a household with preference parameter θ at the beginning of a period in which it belongs to the pool but does not own a house. Then, we have that

$$V_\theta(g,T) = \max_{l\in\{0,1\}} \left\{ \begin{array}{c} l\left(\theta + B(g) - T/H - P(g,T) + \beta EP(g',T')\right) \\ +(1-l)\underline{u} + \beta[\mu EV_\theta(g',T') + (1-\mu)\frac{\underline{u}}{1-\beta}] \end{array} \right\}, \tag{2}$$

where $EP(g',T')$ denotes the expected price of housing next period; i.e.,

$$EP(g',T') = \pi(g)P((1-\delta)g + I(g), cI(g)) + (1-\pi(g))\,P((1-\delta)g, 0), \tag{3}$$

and $EV_\theta(g',T')$ denotes the expected payoff of a household in the pool next period; i.e.,

$$EV_\theta(g',T') = \pi(g)V_\theta((1-\delta)g + I(g), cI(g)) + (1-\pi(g))\,V_\theta((1-\delta)g, 0). \tag{4}$$

Inspecting this problem, it is clear that a household of type θ will choose to reside in the community if

$$\theta + B(g) - T/H - P(g,T) + \beta EP(g',T') \geq \underline{u}. \tag{5}$$

The left hand side of this inequality represents the per-period payoff from locating in the community, assuming that the household buys a house at the beginning of the period and sells it the next. This payoff depends on the preference parameter θ, public good surplus, and the current and future price of housing. The right hand side represents the per period payoff from living elsewhere.

Equilibrium Given an initial state (g,T), the price of housing $P(g,T)$ adjusts to equate demand and supply. The demand for housing is the fraction of households for whom (5) holds. Given the uniform distribution of preferences, this fraction is

$$1 - \frac{\underline{u} - (B(g) - T/H - P + \beta EP(g',T'))}{\overline{\theta}}. \tag{6}$$

[7] It should be stressed that this is just a convenient way of understanding the household decision problem. The equilibrium we study is perfectly consistent with the assumption that the only households selling their homes are those who plan to leave the community.

The supply of housing is, by assumption, perfectly inelastic at H. The equilibrium price of housing therefore satisfies

$$1-\frac{\underline{u}-(B(g)-T/H-P(g,T)+\beta EP(g',T'))}{\overline{\theta}}=H. \tag{7}$$

To characterize the housing market equilibrium, define the present value of public good surplus $S(g,T)$ recursively as follows:

$$S(g,T)=B(g)-T/H+\beta\left[\pi(g)S((1-\delta)g+I(g),cI(g))+(1-\pi(g))S((1-\delta)g,0)\right]. \tag{8}$$

Intuitively, $S(g,T)$ represents the discounted value of future public good surplus for a household who will be living in the community permanently starting in a period in which the community has public good level g and tax obligation T. Note that the recursive definition implies that surplus will depend on the community's entire expected future investment path as defined by the functions $I(g)$ and $\pi(g)$. Then, we have:

Proposition 1 *In equilibrium, those households for whom $\theta\in[(1-H)\overline{\theta},\overline{\theta}]$ choose to reside in the community and those for whom $\theta\in[0,(1-H)\overline{\theta}]$ do not. For households choosing to reside in the community there exists a constant $\kappa(\theta)$ such that*

$$V_{\theta}(g,T)=\kappa(\theta)+S(g,T)-P(g,T), \tag{9}$$

while for households choosing not to reside in the community

$$V_{\theta}(g,T)=\frac{\underline{u}}{1-\beta}. \tag{10}$$

Furthermore, there exists a constant K such that the equilibrium housing price is given by

$$P(g,T)=K+S(g,T). \tag{11}$$

Proof: See Appendix A.

The first part of the proposition tells us that the fraction H of households who choose to reside in the community are those in the pool with the highest preference parameters. This should make good sense intuitively since in all other respects potential residents are identical. The second part gives us expressions for the expected payoffs of the different types of households. These expressions will be useful later in the paper. The final part tells us that the equilibrium price can be expressed as the sum of a constant and the value of public good surplus. Equation (11) implies that the value of future public good levels and tax obligations is fully capitalized into the price of housing and follows from the assumption that the supply of houses is fixed. The constant K is tied down by the requirement that the marginal household with preference $(1 - H)\,\overline{\theta}$ is just indifferent between living and not living in the community.[8]

Equation (11) nicely illustrates the importance of dynamic considerations in understanding capitalization effects of local public goods and amenities. This is because it shows that what is capitalized is the community's entire expected future investment path. In particular, therefore, the implications of current provision levels for future levels will be critically important. The significance of such dynamic considerations is being increasingly recognized in empirical studies. In order to estimate households' willingness to pay to avoid violent crime, Bishop and Murphy (2011) take into account how changes in the current crime rate impact households' expectations concerning the future evolution of the crime rate. They show that ignoring this dynamic effect results in underestimating the extent of capitalization and households' willingness to pay to avoid crime. A similar point emerges in empirical work on environmental clean-up policies. Contrasting the work of Greenstone and Gallagher (2008) and Gamper-Rabindran and Timmins (2013), we see that the net benefits of EPA's Superfund program hinge on assumptions about household expectations for the future evolution of cleanups. While Greenstone and Gallagher find a near-zero capitalization effect of hazardous waste cleanups, Gamper-Rabindran and Timmins find a positive effect. One reason for the difference is that Greenstone and Gallagher fail to distinguish between sites that are cleaned up today and sites that are scheduled for cleanups in the future. Households' discount rate and expectations for the probability of success are likely to be empirically important because the cleanup process often takes a decade or more.

Finally, it should be clear from Proposition 1 that households' equilibrium payoffs and the price of housing would be the same if the investment were financed via a bond issue rather than a tax increase. All that matters is the discounted present value of tax obligations and a policy change which held this constant but altered the future timing of taxes would have no impact on the current price of housing.[9] Ricardian Equivalence therefore holds in this model. Similar

[8] It is straightforward to show that K equals $\kappa((1-H)\,\overline{\theta}) - \underline{u}/(1-\beta)$. To guarantee that housing always has a positive value, it must be the case that the parameters and investment path are such that $\kappa((1-H)\,\overline{\theta}) + S(g,T)$ always exceeds $\underline{u}/(1-\beta)$. We will assume this in what follows.

[9] The future housing price path would be impacted by the choice of debt versus taxes. Suppose, for example, that the cost of investment was financed by issuing one period bonds. Then, while the price of housing in the period after the investment was approved would be the same as under tax finance, the price in the subsequent period when

remarks apply if, once approved, the investment comes on tap over a sequence of future periods rather than all in the next period as assumed here.

3 Optimal public good provision

This section characterizes the evolution of the public good under the assumption that the investment path is chosen by a planner whose objective is to maximize the residents' payoffs. This will establish the normative benchmark departures from which the housing price test is supposed to detect.

Suppose that the current level of the public good is g and the tax obligation is T. From Proposition 1, the residents of the community consist of those households for whom θ lies between $(1-H)\overline{\theta}$ and $\overline{\theta}$. At the time the planner is choosing investment, these households all own houses. Thus, the expected continuation payoff of each of these households next period if the planner implements I units of investment is given by

$$P((1-\delta)g+I,cI)+\mu V_{\theta}((1-\delta)g+I,cI)+(1-\mu)\frac{\underline{u}}{1-\beta} = \mu\kappa(\theta)+(1-\mu)(\frac{\underline{u}}{1-\beta}+K)+S((1-\delta)g+I,cI). \tag{12}$$

To understand the left hand side of (12), consider a home-owning household at the beginning of the next period. The public good level and tax obligation will be $((1-\delta)g+I,cI)$. As noted above, we can assume wlog that the household will sell their house and obtain a payoff $P((1-\delta)g+I,cI)$. With probability μ, they will remain in the set of potential residents and obtain a payoff $V_{\theta}((1-\delta)g+I,cI)$ and with probability $1-\mu$ they will exit the pool and obtain payoff $\underline{u}/(1-\beta)$. The right hand side of (12) follows immediately from equations (9) and (11) of Proposition 1 and tells us that the continuation payoff can be written as the sum of a type-specific constant and the value of public good surplus.

It follows from (12) that choosing investment to maximize resident payoffs is equivalent to maximizing public good surplus.[10] Letting $S^o(g,T)$ denote maximized surplus and using the fact that investment proposals are passed with probability one in an optimal plan (i.e., $\pi(g)\equiv 1$), we

the bonds must be repaid would be lower. This is because taxes must be levied, whereas, with tax finance, the investment is already paid for. However, this is irrelevant for the purposes of this paper which is concerned solely with the immediate impact of an approved investment on housing prices.

[10] This conclusion arises despite the fact that residents may leave the community and thus not get to enjoy the fruits of their investment. The intuition is that, when they leave, residents will sell their homes and the price they get will reflect the future benefits.

know that

$$S^o(g,T) = \max_{I \geq 0} B(g) - T/H + \beta S^o((1-\delta)g + I, cI). \tag{13}$$

Notice that the community's current tax obligation T has no impact on optimal investment because utility is linear in consumption, implying no income effects. Thus, we can denote the surplus maximizing investment rule by $I^o(g)$. Solving for this rule, we obtain:

Proposition 2 *The optimal investment rule is that*

$$I^o(g) = \begin{cases} g^o - (1-\delta)g \ \text{ if } g \leq g^o/(1-\delta) \\ 0 \ \text{ if } g > g^o/(1-\delta) \end{cases}, \tag{14}$$

where the public good level g^o satisfies the dynamic Samuelson Rule

$$HB'(g^o) = c\left[1 - \beta(1-\delta)\right]. \tag{15}$$

Proof: See Appendix A.

Proposition 2 tells us that the optimal investment rule is to get the public good level to g^o as fast as possible and then keep it there. The optimal level g^o satisfies the condition that the sum of one period marginal benefits equals the "one period marginal cost". The latter reflects the fact that investing one unit today saves $c(1-\delta)$ in investment costs tomorrow and these future cost savings have a present value of $\beta c(1-\delta)$.

4 The housing price test

4.1 Non-durable public goods

We begin by reviewing the logic underlying the housing price test in the context of non-durable public goods. The claim to be evaluated is that the housing price response to a small, permanent increase in public good level reveals the efficiency of provision. Suppose therefore that the community currently provides g units of public good per period which generates a tax obligation of cg and consider a small, permanent increase in provision of Δg with associated tax obligation $c\Delta g$.

In our model, we capture this scenario by assuming 100% depreciation ($\delta = 1$), which effectively makes the public good non-durable. In addition, we assume that the investment path is such that the proposed investment is always $g + \Delta g$ and it is always implemented. These assumptions imply that the community's public good level and tax obligation each period will be $(g+\Delta g, c\,(g+\Delta g))$. From (8), public good surplus is

$$S(g+\Delta g, c\,(g+\Delta g)) = \frac{B(g+\Delta g) - c\,(g+\Delta g)\,/H}{1-\beta}. \tag{16}$$

Now let $\Delta P(g, \Delta g)$ denote the difference in housing prices with and without the public good increase.[11] Proposition 1 implies that the price difference is equal to the difference in surplus; that is,

$$\Delta P(g, \Delta g) = S(g+\Delta g, c\,(g+\Delta g)) - S(g, cg). \tag{17}$$

The price difference will therefore be positive if the increase has raised surplus, and negative if not. If the increase is small, then the difference in surplus is approximately equal to the total derivative of surplus multiplied by the increase; that is,

$$S(g+\Delta g, c\,(g+\Delta g)) - S(g, cg) \approx \frac{dS(g, cg)}{d\Delta g}\Delta g. \tag{18}$$

From (16), the total derivative of surplus is

$$\frac{dS(g, cg)}{d\Delta g} = \frac{B'(g+\Delta g) - c/H}{1-\beta}. \tag{19}$$

The optimal level of the public good g^o satisfies the static Samuelson Rule that the sum of marginal benefits $HB'(g)$ equals the marginal cost c. From (19) this implies that the derivative of surplus is zero at the optimal level; i.e., $dS(g^o, cg^o)/d\Delta g = 0$. Thus, since surplus is concave in Δg, we have that

$$\frac{dS(g, cg)}{d\Delta g}\Delta g \gtreqless 0 \Longleftrightarrow g \lesseqgtr g^o. \tag{20}$$

From (17) and (18), therefore, the housing price response to a small, permanent increase in the public good is, to a first approximation, positive if g is less than g^o and negative if g exceeds g^o.

The assumption that the increase in public good level is small is important to this logic. Nonetheless, the housing price test is still informative when this assumption is not satisfied. In this case, the public good level *with* the increase (i.e., $g+\Delta g$) must be distinguished from the level *without* (i.e., g). Since surplus is strictly concave in Δg, it follows from (17) that

$$\frac{dS(g+\Delta g, c\,(g+\Delta g))}{d\Delta g}\Delta g < \Delta P(g, \Delta g) < \frac{dS(g, cg)}{d\Delta g}\Delta g. \tag{21}$$

[11] That is, $\Delta P(g, \Delta g) = P(g+\Delta g, c\,(g+\Delta g)) - P(g, cg)$.

A non-negative price difference therefore signals that $dS(g, cg)/d\Delta g$ is positive and hence the public good level without the increase is below optimal. By contrast, a non-positive difference signals that $dS(g + \Delta g, c(g + \Delta g))/d\Delta g$ is negative and hence the level with the increase is too high.

4.2 Durable public goods

We now explore whether a similar logic implies that the housing price response to an investment sheds light on the efficiency of durable public good provision. Suppose the current public good level is g and the proposed investment is $I(g)$. Next period's public good level and tax obligation will be $((1-\delta)g + I(g), cI(g))$ if the investment is implemented and $((1-\delta)g, 0)$ if not. Let the difference in the price of housing that would prevail next period with and without the investment be denoted $\Delta P(g, I(g))$.[12] Proposition 1 implies that the price difference equals the difference in surplus; that is,

$$\Delta P(g, I(g)) = S((1-\delta)g + I(g), cI(g)) - S((1-\delta)g, 0). \tag{22}$$

A positive price difference implies that the investment has increased surplus, while a negative difference implies that surplus has decreased.

If we assume the investment is small, we can approximate the change in surplus in a similar manner as in (18). However, this is not a tenable assumption for a durable good subject to depreciation. After all, just to maintain public good levels, it will be necessary to have investment sufficient to offset depreciation. Thus, unless the depreciation rate is infinitessimal, $I(g)$ cannot be small in steady state. But, as in the non-durable case, this is not a significant problem. We just need to distinguish the level of public good with the investment (i.e., $(1-\delta)g + I(g)$) and the level without (i.e., $(1-\delta)g$). Assuming that surplus is strictly concave in I, we have that[13]

$$\frac{dS((1-\delta)g + I(g), cI(g))}{dI} I(g) < \Delta P(g, I(g)) < \frac{dS((1-\delta)g, 0)}{dI} I(g). \tag{23}$$

[12] This price difference corresponds to what Cellini, Ferreira, and Rothstein (2010) refer to in their empirical work as the "intent-to-treat" (ITT) effect of the investment on housing prices. It represents the reaction of housing prices to the investment *assuming that all future investment decisions will be made according to the community equilibrium*. They also discuss a "treatment on the treated" (TOT) effect which is the hypothetical reaction of housing prices to the investment *assuming there were no future investments*. As will be pointed out below in footnote #14, this paper's critique also applies to this latter measure.

[13] Note that $dS((1-\delta)g + I, cI)/dI = \partial S((1-\delta)g + I, cI)/\partial g + c\partial S((1-\delta)g + I, cI)/\partial T$. However, $\partial S((1-\delta)g + I, cI)/\partial T$ is equal to $-1/H$. Thus, if $\partial^2 S((1-\delta)g + I, cI)/\partial g^2$ is negative, $dS((1-\delta)g + I, cI)/dI$ is decreasing in I.

Moreover, since surplus is linear in the tax obligation T, we know that $dS((1-\delta)g+I(g),cI(g))/dI$ must equal $dS((1-\delta)g+I(g),0)/dI$. Thus, if it were the case that the optimal level of the public good g^o satisfied the first order condition that $dS(g^o,0)/dI$ equals zero, a non-negative price difference would signal that the public good level without the investment is less than the optimal level g^o, while a non-positive price difference would signal that the level with investment is greater than g^o. This is the form of the housing price test that is relevant in the durable good context.

4.2.1 The difficulty

The difficulty with the foregoing analysis lies in the assumption that the socially optimal public good level g^o satisfies the first order condition that $dS(g^o,0)/dI$ equals zero. This is true for the socially optimal surplus function $S^o(g,T)$ but the *equilibrium* public good surplus function $S(g,T)$ will not in general equal the *optimal* surplus function $S^o(g,T)$. The former assumes that future investments are governed by the investment path described by $I(g)$ and $\pi(g)$, while the latter assumes future decisions are made optimally. As is well known from the theory of the second best, the fact that some decisions are not optimal typically means that the rules governing the decisions that can be optimized will change.

To see the difficulty formally, note from (8) that the derivative of surplus with respect to investment is

$$\frac{dS((1-\delta)g+I,0)}{dI} = [B'((1-\delta)g+I)-c/H]+$$
$$\beta\left[\begin{array}{c}\frac{d\pi(\cdot)}{dg}\left\{S((1-\delta)\left[(1-\delta)g+I\right]+I(\cdot),cI(\cdot))-S((1-\delta)\left[(1-\delta)g+I\right],0)\right\}\\ +\pi(\cdot)\left\{\frac{\partial S((1-\delta)[(1-\delta)g+I]+I(\cdot),cI(\cdot))}{\partial g}[(1-\delta)+\frac{dI(\cdot)}{dg}]-c\frac{dI(\cdot)}{dg}/H\right\}\\ +(1-\pi(\cdot))\frac{\partial S((1-\delta)[(1-\delta)g+I],0)}{\partial g}(1-\delta)\end{array}\right], \tag{24}$$

where to compact notation $\pi(\cdot)$ denotes $\pi((1-\delta)g+I)$ and $I(\cdot)$ denotes $I((1-\delta)g+I)$. The term in square brackets on the top line of (24) measures the immediate consequences of an increase in investment on surplus: public good benefits go up as do taxes. The second term measures the future consequences and these are evidently quite complicated. In particular, account must be taken of how an increase in current investment will impact next period's proposed investment (i.e., $dI(\cdot)/dg$) and also the probability that this is implemented (i.e., $d\pi(\cdot)/dg$).

A necessary and sufficient condition for $dS(g^o,0)/dI$ to equal zero is that the second term in

(24) equals $\beta(1-\delta)c/H$ when evaluated at g^o. For only then will it be the case that

$$\frac{dS(g^o,0)}{dI} = B'(g^o) - c[1-\beta(1-\delta)]/H. \tag{25}$$

Given the dynamic Samuelson Rule (15), this is required for $dS(g^o,0)/dI$ to equal zero. Intuitively, when the second term in (24) equals $\beta(1-\delta)c/H$, the future consequence of a marginal increase in current investment is just the discounted value of a compensating decrease in the amount of investment made next period. With depreciation, a unit of investment this period creates $1-\delta$ of a unit next period and so a compensating decrease would be $1-\delta$. This would save each resident $c(1-\delta)/H$ in taxes and this has a present value of $\beta(1-\delta)c/H$. Under an optimal investment plan, the second term in (24) will indeed equal $\beta(1-\delta)c/H$. To see this, note from (14) that $dI(\cdot)/dg$ is equal to $-(1-\delta)$ and, since investment proposals are implemented with probability one, $\pi(\cdot)$ is equal to one and $d\pi(\cdot)/dg$ is equal to zero. In equilibrium, however, there is no reason to believe that the future consequences of an increase in investment will be so simple.

4.2.2 When and how will the housing price test fail?

The housing price test can fail in two ways: falsely predicting under or over-provision. It *falsely predicts under-provision* at public good level g if the housing price difference $\Delta P(g, I(g))$ is non-negative, but the level of the public good without the investment $(1-\delta)g$ exceeds the socially optimal level g^o. Conversely, it *falsely predicts over-provision* at g if the housing price difference is non-positive, but the level of the public good with the investment $(1-\delta)g + I(g)$ is smaller than the socially optimal level g^o.

The potential for the housing price test to fail is present whenever $dS(g^o,0)/dI$ is not equal to zero, but whether failure occurs depends on the details of the community's investment path. When $dS(g^o,0)/dI$ is positive, the potential is for the test to falsely predict under-provision. To understand this, note that for public good levels g such that $(1-\delta)g$ is equal to or marginally bigger than g^o, the fact that $dS(g^o,0)/dI$ is positive implies that the surplus with the associated investment level $S((1-\delta)g+I(g), cI(g))$ could be greater than or equal to $S((1-\delta)g, 0)$. This would imply a non-negative price difference and a false prediction of underprovision. However, whether surplus with investment exceeds that without investment, depends on how big the investment level $I(g)$ is. It could be sufficiently large that $S((1-\delta)g + I(g), cI(g))$ is actually smaller than $S((1-\delta)g, 0)$, in which case the price difference would be negative. In that scenario, the housing price test correctly predicts that $(1-\delta)g + I(g)$ is higher than optimal.

Conversely, when $dS(g^o,0)/dI$ is negative, the potential is for the test to falsely predict over-provision. To see this, note that for public good levels g such that $(1-\delta)g+I(g)$ is equal to or marginally smaller than g^o, the fact that $dS(g^o,0)/dI$ is negative implies that surplus with investment $S((1-\delta)g+I(g),cI(g))$ could be smaller than or equal to $S((1-\delta)g,0)$. This would imply a non-positive price difference and a false prediction of over-provision. However, again, whether surplus with investment is smaller than that without depends on the size of the investment level $I(g)$. If it is sufficiently large, it could be that $S((1-\delta)g+I(g),cI(g))$ is greater than $S((1-\delta)g,0)$ in which case the price difference is positive and the housing price test correctly predicts that $(1-\delta)g$ is smaller than optimal.

At a general level, it is also unclear which *type* of failure (i.e., falsely predicting under or over-provision) is more likely to arise since the likely sign of $dS(g^o,0)/dI$ is unclear. The sign of $dS(g^o,0)/dI$ is positive (negative) when the second term in (24) evaluated at g^o exceeds (falls short of) $\beta(1-\delta)c/H$. Differencing these two expressions, we obtain

$$\beta\left[\begin{array}{c} \frac{d\pi(\cdot)}{dg}\left(S((1-\delta)g^o+I(\cdot),cI(\cdot))-S((1-\delta)g^o,0)\right) \\ +\pi(\cdot)\frac{\partial S((1-\delta)g^o+I(\cdot),cI(\cdot))}{\partial g}\left(1-\delta+\frac{dI(\cdot)}{dg}\right)-\frac{c}{H}\left(1-\delta+\pi(\cdot)\frac{dI(\cdot)}{dg}\right) \\ +\left(1-\pi(\cdot)\right)\frac{\partial S((1-\delta)g^o,0)}{\partial g}(1-\delta) \end{array}\right]. \tag{26}$$

The sign of this expression determines the sign of $dS(g^o,0)/dI$. The sign of the first term is unclear because there is no natural assumption to make on the sign of $d\pi(\cdot)/dg$. While *ceteris paribus* having a higher level of the public good might be expected to reduce the probability of an investment proposal being implemented, it might also reduce the size of the proposed investment, so the net effect is uncertain. The sign of the second term is ambiguous because again there is no natural assumption to make on how $dI(\cdot)/dg$ will compare with $-(1-\delta)$: that is, will an increase in public good level lead to a more or less than compensating adjustment in new investment? Moreover, even if that issue were resolved, the relative magnitudes of $\partial S/\partial g$ and c/H are not obvious.[14]

To obtain a deeper understanding of when and how the housing price test might fail, we need

[14] Suppose that we instead evaluated public good provision using the hypothetical housing price response to the investment assuming there were no future investments (the TOT effect discussed in footnote #12). With no future investments, public good surplus would be $S(g,T)=\sum_{t=0}^{\infty}\beta^t B(g(1-\delta)^t)-T/H$. It is easily verified that with this surplus function, $dS(g^o,0)/dI$ exceeds zero. Intuitively, if no investment will take place in the future, the optimal public good level today will be much larger than g^o. As a consequence, a positive housing price response would not imply the public good level without investment was below optimal.

more information about the community's investment path. The next section proposes an explicit model of the community's political process and develops its implications for the community's investment path. It then uses this information to scrutinize the performance of the housing price test.

5 Public good provision with a budget-maximizing bureaucrat

To investigate the potential problems with the housing price test more concretely, we now analyze what it tells us when the community's investment path is determined by the interaction between a budget-maximizing bureaucrat and the residents. More specifically, we assume that the investment proposal $I(g)$ and the probability it is implemented $\pi(g)$ are generated by an extended version of the model proposed by Barseghyan and Coate (2014). These authors study the provision of a durable public good under the assumptions that investment is chosen by a policy-maker but must be approved by the residents of the community in a referendum. The policy-maker is assumed to be a "budget-maximizing bureaucrat", caring only about the level of the good and not its cost.[15] We extend their model in two ways. First, we assume that residents vote with noise in referenda. This makes the referendum uncertain and means that the bureaucrat's investment proposals may not be implemented in equilibrium. This both enrichens the model and makes it consistent with the empirical strategy of Cellini, Ferreira, and Rothstein (2010) which requires that proposed investments sometimes pass and sometimes fail. Second, we allow the bureaucrat to have strictly concave preferences over the public good. This assumption makes no difference in the world of Barseghyan and Coate, because the bureaucrat faces no risk. However, in the extended model with uncertain referendum outcomes, the bureaucrat's risk aversion influences his proposed investment in interesting ways.

5.1 The model with a budget-maximizing bureaucrat

Imagine that a bureaucrat (or bureaucrats - for example, a school board or a city council) manages the provision of the public good for the community. In any period, the bureaucrat can propose investment but, to be implemented, the proposal must be approved by a majority of the residents. The bureaucrat decides how much investment to propose after the housing market has cleared

[15] The model is essentially a dynamic version of Romer and Rosenthal's agenda setter model.

and new residents have moved into the community. The bureaucrat cares only about the level of the public good and not the tax cost of financing it. In a period in which the public good level is g, the bureaucrat obtains a per-period utility $u(g)$ where u is increasing and concave. Like the residents, the bureaucrat discounts future payoffs at rate β.

Residents' voting decisions Suppose that the current level of the public good is g and the bureaucrat has proposed investment I. Consider a resident household with preference parameter θ. Following the literature on probabilistic voting, the household votes in favor of the bureaucrat's proposal if the difference in his continuation payoff with and without the proposed investment exceeds the value of a voting preference shock. Formally, this corresponds to:

$$\begin{gathered} P((1-\delta)g+I,cI)+\mu V_\theta((1-\delta)g+I,cI)+(1-\mu)\tfrac{u}{1-\beta} \geq \\ \left[P((1-\delta)g,0)+\mu V_\theta((1-\delta)g,0)+(1-\mu)\tfrac{u}{1-\beta}\right]+\epsilon_\theta+\eta. \end{gathered} \tag{27}$$

On the left hand side of the inequality is the expected continuation payoff of the household next period if the investment is implemented. On the right hand side, the term in the square bracket is the expected continuation payoff next period without the investment. The additional term $\epsilon_\theta+\eta$ captures the household's bias in favor of voting against the bureaucrat's proposal. This bias, which is unrelated to the details of the bureaucrat's proposal, consists of an idiosyncratic component ϵ_θ and an aggregate component η.[16] The idiosyncratic component reflects the individual household's bias in favor of voting against the bureaucrat's proposal and is assumed to be the realization of a random variable uniformly distributed on $[-\epsilon,\epsilon]$ where ϵ is a large positive number. The aggregate component impacts all households in a uniform way and is assumed to be the realization of a random variable normally distributed with mean 0 and variance σ^2.

Using Proposition 1, we can write the difference in the resident's continuation payoffs as the difference in public goods surplus. Thus, (27) can be rewritten as

$$S((1-\delta)g+I,cI)-S((1-\delta)g,0)\geq\epsilon_\theta+\eta, \tag{28}$$

where the surplus function is as defined in (8) (with $I(g)$ and $\pi(g)$ the functions associated with the equilibrium). Given the assumed uniform distribution of the shock ϵ_θ, the fraction of residents

[16] This bias is a simple way to introduce uncertainty into the outcome of the referendum. It is a modelling trick that is used extensively in the political economy literature (see Persson and Tabellini 2000 for references and discussion). Perhaps the best way to think of the bias in this context, is that it represents the voter's pure utility from voting for or against the bureaucrat's proposal. This may reflect ideology or psychological considerations such as spite or generosity of spirit.

voting in favor of the proposal given the aggregate shock η is[17]

$$\frac{S((1-\delta)g+I,cI)-S((1-\delta)g,0)-\eta}{2\epsilon}+\frac{1}{2}. \tag{29}$$

Given that implementation requires the approval of a majority of residents, the proposal is implemented when this fraction exceeds 1/2. The probability that the proposal is approved is therefore the probability that the aggregate shock η is less than the surplus difference. Given the assumed normal distribution of the shock η, this probability is given by

$$\varphi(g,I)=\Phi\left(S((1-\delta)g+I,cI)-S((1-\delta)g,0)\right) \tag{30}$$

where Φ denotes the cumulative distribution function of the normal distribution. This expression reveals a very simple relationship between the probability of a proposal being approved and the surplus it generates for the residents.

The bureaucrat's investment proposal Let the bureaucrat's value function be denoted by $U(g)$. Then, when the public good level is g, the bureaucrat will choose an investment proposal

$$I(g)=\arg\max\left\{\begin{array}{c} u(g)+\beta\left[\varphi(g,I)U((1-\delta)g+I)+(1-\varphi(g,I))\,U((1-\delta)g)\right] \\ s.t.\ I\geq 0\end{array}\right\}. \tag{31}$$

The probability the proposal will be implemented, $\pi(g)$, will then satisfy the equality $\pi(g)=\varphi(g,I(g))$. The bureaucrat's value function will satisfy the functional equation

$$U(g)=u(g)+\beta\left[\pi(g)U((1-\delta)g+I(g))+(1-\pi(g))\,U((1-\delta)g)\right]. \tag{32}$$

The equilibrium investment path The investment path $I(g)$ and $\pi(g)$ is an *equilibrium investment path* if, when the public good surplus function $S(g,T)$ is defined by (8), the probability of approval $\varphi(g,I)$ is defined by (30), and the bureaucrat's value function is defined by (32), the investment proposal function $I(g)$ solves problem (31) and the implementation probability function $\pi(g)$ equals $\varphi(g,I(g))$.

Solving for the equilibrium investment path Solving for the equilibrium investment path is not a straightforward exercise. As shown by Barseghyan and Coate (2014), the interaction

[17] We assume that ϵ is sufficiently large that $\left(S((1-\delta)g+I,cI)-S((1-\delta)g,0)-\eta\right)/2\epsilon+1/2$ lies in the $[0,1]$ interval.

between the bureaucrat and residents is quite complex. They obtain an analytical solution for the equilibrium only for specific utility functions. More generally, they use numerical methods to find the equilibrium. Our setting is more complicated because of the uncertainty in the behavior of residents at the ballot box. Accordingly, we solve for equilibrium investment paths numerically. In addition, our focus is on finding examples which illustrate different possible outcomes under the housing price test. Thus, we do not attempt to provide a comprehensive picture of what equilibrium investment paths look like in this environment.

The computational procedure involves solving for the fixed points as characterized by the value functions $U(g)$ and $S(g,T)$ through value function iterations.[18] Since the bureaucrat's objective function in problem (31) is not necessarily concave over the entire domain, we solve the problem through a discrete grid search method instead of solving the system of equations implied by the first order conditions. More specifically, we first specify initial guesses for value functions $U(g)$ and $S(g,T)$ over a grid $\mathcal{G}$ of public good levels. Based on these initial guesses, for each point g on grid $\mathcal{G}$, we evaluate the bureaucrat's objective as specified in problem (31) at all possible points over the grid $\mathcal{G}$ and find the level $g'(g)$ that achieves the maximum value subject to the constraint that $g'(g) \geq (1-\delta)g$. In this way, we find the bureaucrat's investment proposal $I(g) = g'(g) - (1-\delta)g$. Using the newly obtained investment policy function $I(g)$ as well as the recursive expressions for public good surplus (equation (8)) and the bureaucrat's value function (equation (32)), we generate new guesses for the value functions $U(g)$ and $S(g,T)$. Using the newly generated value functions as the initial guesses, we simply repeat the above procedure until convergence of the value functions is achieved. The equilibrium investment path is then that associated with these value functions.

In terms of parameterization, we assume that the bureaucrat's utility function $u(g)$ has the functional form $g^{1-\gamma}/(1-\gamma)$ and that the residents' public good benefit function $B(g)$ has the functional form $\sqrt{g}$. We also assume that the discount factor β equals 0.9, the depreciation rate δ equals 0.1, the per unit investment cost c is 1, and housing supply H equals 0.8. We obtain our different examples by varying the standard deviation of the preference shock σ and the bureaucrat's risk aversion parameter γ.

[18] To be more accurate, we iterate over modified versions of the value functions $U(g)$ and $S(g,T)$ where the current public good level g is the only state variable. Appendix B provides a detailed description of the solution procedure.

5.2 The housing price test with a budget-maximizing bureaucrat

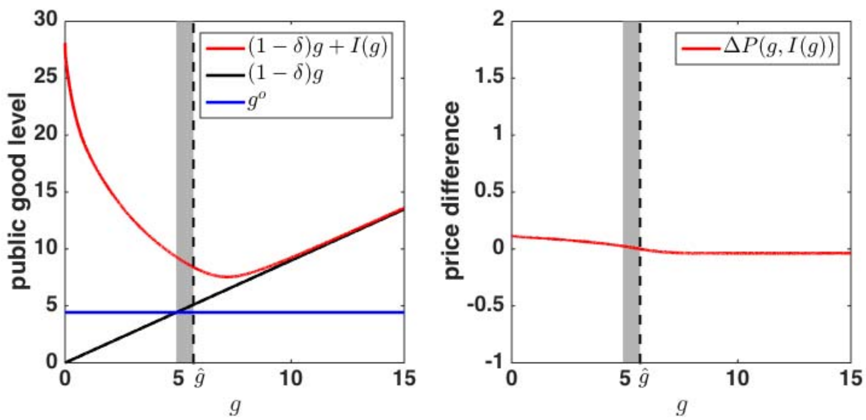
We now present our three examples which illustrate the different possible outcomes under the housing price test.

5.2.1 Example 1: False prediction of under-provision

We begin with an example in which the housing price test falsely predicts under-provision. For this example, we make the bureaucrat risk neutral by setting the risk aversion parameter γ equal to 0. We also set the standard deviation of the aggregate voting shock σ equal to 0.05. This low variance assumption means that the outcome of the referendum basically hinges on the comparison of public good surplus with and without investment. Thus, there is a very high probability of the investment proposal passing if it raises public good surplus and a very low probability of passage if it reduces surplus. Given this, we would expect the equilibrium investment proposal to be quite similar to that in the equilibrium analyzed in Barseghyan and Coate (2014) (who assume no noise in residents' voting decisions). Moreover, given that these authors showed that in their equilibrium the surplus maximizing public good level exceeds the socially optimal level, we would expect $dS(g^o, 0)/dI$ to be positive. This creates the potential for the housing price test to falsely predict under-provision.

Figure 1 plots the equilibrium public good level and housing price difference as a function of the current public good level. The left panel describes the equilibrium public good level. The red line describes the level assuming the bureaucrat's proposed investment is approved and the black line describes the level in the case of rejection. As expected, the behavior of the equilibrium investment proposal $I(g)$ is very similar to that in the equilibrium characterized by Barseghyan and Coate. When the current public good level is low, investment proposals are large because the bureaucrat exploits the fact that failing to invest is very unappealing to residents. This is so not only because the residents will have to live with an even smaller public good level next period because of depreciation but also because the bureaucrat will exploit next period's depreciated level to propose an even larger investment. As the current public good level increases, the prospect of failing to invest becomes less daunting and residents' willingness to pass large proposals decreases. The bureaucrat responds by making smaller investment proposals. These proposals eventually converge to zero and so the equilibrium public good level with investment gradually approaches the depreciated level as g increases. The right panel describes the housing price difference. This

Figure 1: False prediction of under-provision



difference starts out positive, but as the current public good level increases, it decreases and eventually turns negative.

Now consider the validity of the housing price test. In both panels, the vertical dashed line denotes the public good level $\widehat{g}$ where the housing price difference $\Delta P(g, I(g))$ hits zero. The blue line in the left panel depicts the socially optimal level of the public good g^o. The housing price test falsely predicts under-provision for the range of public good levels described by the gray region. For any public good level g within this region the price difference in the right panel is positive and hence the housing price test predicts that the optimal public good level is above $(1-\delta)g$. The left panel, however, reveals that g^o lies below $(1-\delta)g$.

To further understand why the housing price test falsely predicts under-provision, recall expression (26). When this is positive, we know that $dS(g^o, 0)/dI$ will be positive. When we numerically compute the relevant derivatives and evaluate expression (26), we indeed find it to be positive. This positive sign reflects the sum of various offsetting effects. On the one hand, $d\pi(g^o)/dg$ is negative, which indicates that a marginal increase in the public good at the socially optimal level decreases the probability the equilibrium investment proposal passes. Since we know from the right panel of Figure 1 that the public good surplus difference evaluated at g^o is also positive, the term on the top line is negative. The sum of the remaining terms, however, is positive and overwhelms the negative top line. This reflects the fact that a marginal increase in the public good at the socially optimal level leads to a large decrease in the equilibrium investment proposal so that $dI(g^o)/dg$ is much more negative than $-(1-\delta)$. Given that the difference in the red and black lines is the investment level, this is clear from the left panel of Figure 1. Intuitively, this reflects the benefit to residents of increasing the public good level to reduce exploitation by the bureaucrat. This, combined with the fact that $\partial S/\partial g$ is smaller than c/H, explains why the sum of the remaining terms is positive.

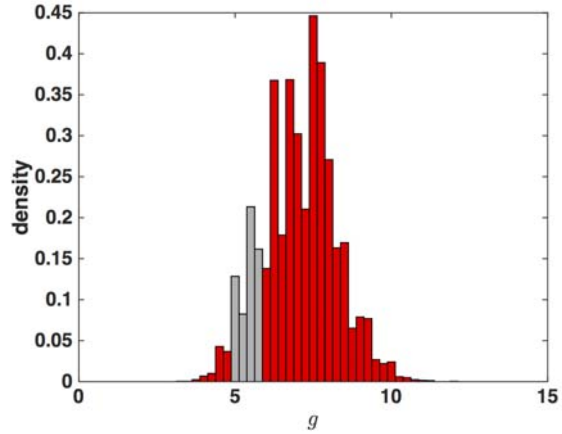
Figure 1 shows that there exist a range of public good levels at which the housing price test falsely predicts under-provision. A natural question to ask is how likely these public good levels are to arise? In particular, are they likely to be encountered only in the transition to the steady state or will they be encountered in the long run? To answer this question, we compute the stationary distribution of public good levels implied by the equilibrium.[19] A distribution of public good levels will be observed in the long run because of the uncertainty in the passage of the bureaucrat's investment proposals. Figure 2 plots this stationary distribution. Observe that the range of public good levels at which the housing price test falsely predicts under-provision lies in the support of this distribution. This means that, even in the long run, the housing price test will fail with non-trivial probability. This said, if the test is applied to a randomly drawn public good level from the stationary distribution, then failure is less likely than success.

5.2.2 Example 2: False prediction of over-provision

We now provide an example in which the housing price test falsely predicts over-provision. Creating this example requires more ingenuity. We make the bureaucrat highly risk averse by setting his risk aversion parameter γ equal to 5. This makes him considerably more risk averse than the residents. In addition, to face the bureaucrat with significant uncertainty, we increase the randomness of referendum outcomes by setting the standard deviation of the aggregate voting

[19] To obtain the stationary distribution, we use the following procedure. Starting with an arbitrary initial public good level g_0, we simulate the economy 10000 times. Each simulation is associated with a sequence of aggregate voting shocks $\{\eta_t\}$ drawn from the normal distribution. For each simulation and any period t, we use the equilibrium investment proposal function $I(g)$ and the equilibrium probability of implementation $\pi(g)$ repeatedly to generate the associated sequence of public good levels $\{g_\tau\}_{\tau=0}^{t}$. To understand how this is done, suppose the public good level in period $\tau-1$ is $g_{\tau-1}$. Then, the public good level in period τ is $(1-\delta)g_{\tau-1}+I(g_{\tau-1})$ if the investment proposal passes and $(1-\delta)g_{\tau-1}$ if it does not. Whether the proposal passes is determined by the period τ shock η_τ. For any period t, if we aggregate over all the simulations, we obtain a cross-sectional distribution of public good levels. We keep increasing t until the public good distribution associated with t is the same as the distribution associated with $t-1$. At that point, we have found the stationary distribution. Notice that in general, the stationary distribution obtained may depend on the initial public good level g_0. However, we find numerically that the same stationary distribution emerges for different initial public good levels suggesting that, in equilibrium, the distribution of public good levels converges to a unique stationary distribution.

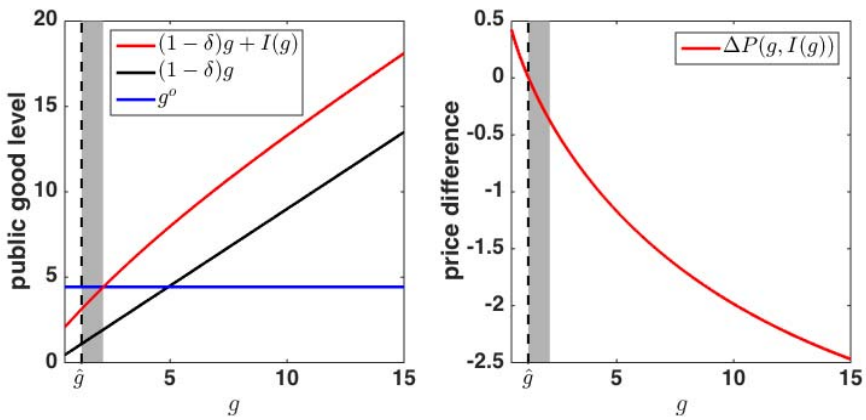
Figure 2: Stationary distribution



shock σ equal to 5. In this setting, we would expect the bureaucrat to be highly conservative in his investment proposals at low levels of the public good. On the other hand, at higher levels of the public good, the bureaucrat may make larger proposals in the hope of taking advantage of the randomness in referendum outcomes. This latter effect may lead residents to prefer to keep the public good level low to constrain the bureaucrat. This could cause $dS(g^o, 0)/dI$ to be negative, creating the potential for the housing price test to falsely predict over-provision.

Figure 3 plots the equilibrium public good level and housing price difference as a function of the current public good level. In the left panel, the red line describes the equilibrium public good level assuming the bureaucrat's proposed investment is approved and the black line describes the level in the case of rejection. The equilibrium investment proposal is driven by two forces. On one hand, the high uncertainty of voting outcomes implies that the chances of proposals being passed by residents 'mistakenly' is quite high. At high current public good levels, this, together with the relatively small utility loss from proposal failure due to high risk aversion, leads the bureaucrat to make large proposals. On the other hand, at low current public good levels, the high risk aversion makes the bureaucrat reluctant to risk failure in the referendum resulting in low investment proposals. Under our parameterization, this second force drives the public good level with investment below the optimal level. From the right panel, the housing price difference starts out positive but, as the current public good level increases, it decreases and eventually turns negative.

Figure 3: False prediction of over-provision

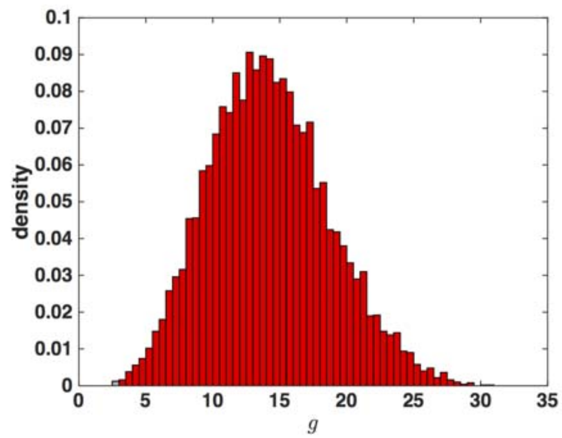


Now consider the validity of the housing price test. In both panels, the vertical dashed line denotes the public good level $\widehat{g}$ where the housing price difference $\Delta P(g, I(g))$ is zero. The blue line in the left panel depicts the socially optimal level of the public good g^o. The housing price test falsely predicts over-provision for the range of public good levels described by the gray region. For any public good level g in this region the price difference in the right panel is negative and hence the housing price test predicts that the optimal public good level should be below $(1-\delta)g + I(g)$. The left panel, however, reveals that g^o lies above $(1-\delta)g + I(g)$.

To get further insight into what is going on in this example consider again expression (26). When this is negative, $dS(g^o, 0)/dI$ will be negative. This is indeed what we find when we numerically compute the derivatives. Again, the negative sign reflects the sum of offsetting forces. The top line is positive but the remaining terms sum to a negative amount which outweighs the top line. The top line is positive because $d\pi(g^o)/dg$ is negative and, from the right panel of Figure 3, the public good surplus difference evaluated at g^o is also negative. The remaining terms sum to a negative amount because $dI(g^o)/dg$ is positive and $\partial S/\partial g$ is smaller than c/H. The positive sign of $dI(g^o)/dg$ is clear from the left panel of Figure 3 and reflects the fact that a marginal increase in the public good at the socially optimal level induces the bureaucrat to make a bolder proposal.

Again, it is interesting to ask whether the range of public good levels identified in Figure 3

Figure 4: Stationary distribution



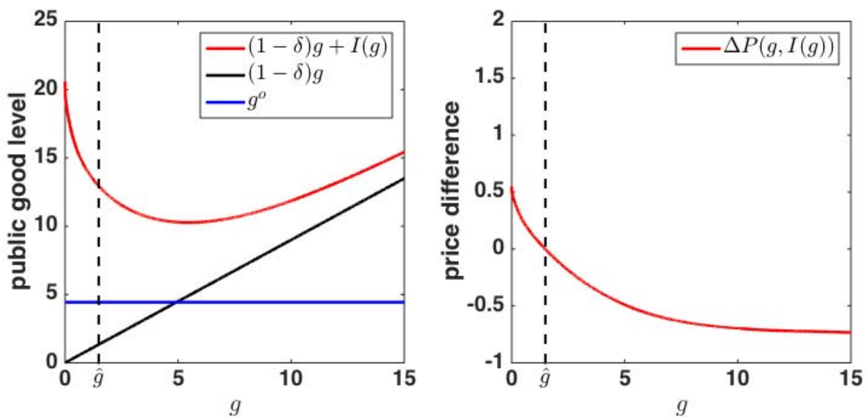
will be encountered in the long run. Figure 4 plots the stationary distribution of the public good in this example. As indicated by the gray color, the housing price test failure occurs at the very left tail of the stationary distribution. Even though the chances of such failure is relatively low, it does occur with positive probability in the long run. In addition, of course, failure may happen in the transition path to the stationary distribution if the economy starts with a low level of public good.

5.2.3 Example 3: No false predictions

Finally, we provide an example in which the housing price test does not fail. In this example, we assume the bureaucrat is risk neutral ($\gamma = 0$) and that there is moderate level of voting uncertainty ($\sigma = 1$). Figure 5 describes the relevant information for this case. In the left panel, we see that the equilibrium public good level with investment always stays above the optimal level. At low levels of g, the bureaucrat proposes high levels of investment. As g increases, the proposed level of investment decreases yet stays positive. In the right panel, we see that the housing price difference is decreasing in the public good level g, starting off positive but soon turning negative.

Now consider the validity of the housing price test. As usual, in both panels, the vertical dashed line denotes the public good level $\widehat{g}$ where the housing price difference is zero and, in the left panel, the blue line depicts the socially optimal level of the public good g^o. Observe that for public good levels less than or equal to $\widehat{g}$ where the housing price difference is non-negative,

Figure 5: No false predictions



the housing price test correctly predicts that $(1-\delta)g$ is below the optimal level. For public good levels greater than or equal to $\widehat{g}$ where the housing price difference is non-positive, the housing price test correctly predicts that $(1-\delta)g + I(g)$ is above the optimal level.

5.2.4 Summary

The examples analyzed in this section illustrate that the housing price test can falsely predict both under-provision and over-provision. They also illustrate, however, that it is perfectly possible for the test to perform flawlessly even in a world in which there are large differences between equilibrium and optimal provision. All of this leaves us with a sense of unease regarding the application of the test. We cannot be sure it will provide inaccurate predictions, but we cannot be sure that it will not. Moreover, if it does provide inaccurate predictions, we cannot be sure of the direction of the bias.

6 An adaptive expectations justification

The analysis so far has been conducted under the standard economic assumption that residents have rational expectations concerning the future evolution of policies in their community. Thus, they understand the dynamic environment they are in and correctly predict how investment in the public good will evolve over time. This is clear in the game-theoretic budget-maximizing bureaucrat example of Section 5. In the general model of Section 2, the assumption is reflected in the idea that the functions $\pi(g)$ and $I(g)$ represent accurate predictions about the community's investment path. This section relaxes this assumption and interprets the functions $\pi(g)$ and $I(g)$ as simply representing residents' beliefs about what will happen. A simple adaptive expectations model is adopted and is shown to provide one way of rationalizing the housing price test.

Specifically, suppose that residents expect that whatever level of public good they observe in the community at the beginning of a period will be maintained indefinitely. Thus, they observe the current quantity and quality of school facilities, say, and just assume they are at steady state levels. Formally, this assumption means that when the state is g, residents expect that investment $I(g)$ will equal δg and the probability of passing this investment $\pi(g)$ is equal to one. These rules imply that residents believe that the present value of public good surplus is given by

$$S(g,T) = B(g) - T/H + \frac{\beta}{1-\beta}\left[B(g) - c\delta g/H\right]. \tag{33}$$

It then follows that the derivative of surplus with respect to investment is

$$\frac{dS((1-\delta)g+I,0)}{dI} = \frac{B'((1-\delta)g+I) - c[1-\beta(1-\delta)]/H}{1-\beta}. \tag{34}$$

In particular, therefore, it is the case that $dS(g^o,0)/dI$ is equal to zero. Accordingly, the housing price test will work as advertised. The intuition is straightforward: a successful investment is interpreted as creating a permanent increase in public good levels and hence we are back in a world where the standard logic applies.

In our judgement, this represents the best way to justify the housing price test. But still there is an important caveat. Observe that

$$\frac{dS((1-\delta)g+I,cI)}{dI} = \left(\frac{1}{1-\beta}\right)\frac{dS^o((1-\delta)g+I,cI)}{dI}. \tag{35}$$

The multiplicative factor $1/(1-\beta)$ means that the change in housing prices over-estimates the willingness to pay for investments. Intuitively, this reflects the fact that, under adaptive expectations, residents interpret a small increase in investment as signaling a permanent increase in the level of the public good. If the good is under-provided, this permanent increase will have a considerably higher value to residents than a temporary one.

7 Conclusion

The theoretical justification for using the housing price response to public investment to evaluate the efficiency of local durable public good provision is based on an understanding of how housing prices respond to permanent changes in local non-durable public good provision. There are clear conceptual differences between an investment in a durable public good and a permanent increase in a non-durable public good. In a framework that accounts for these differences, we have shown that the logic underlying the non-durable case translates to the durable case if the socially optimal level of the public good maximizes the surplus residents are expected to receive from provision *in equilibrium*; that is, when future investment follows the path determined by the community's political decision-making process. However, the theory of the second best teaches us that there is no reason to expect this to be the case.

When the socially optimal level of the public good does not maximize the residents' equilibrium surplus, the housing price response to investment can provide misleading information concerning the efficiency of provision. Under the assumption that the community's investment path is determined by the interaction between a budget-maximizing bureaucrat and the residents, we have provided examples in which using the housing price response can falsely predict that public goods are under-provided, falsely predict they are over-provided, or perform without error. These examples leave us uncomfortable with the technique. We cannot be sure it will fail, but we cannot be sure that it will not. Moreover, if it does fail, the direction of its bias is uncertain.

We have pointed out that one way of justifying the technique is to assume that residents have adaptive expectations, believing that whatever provision level that currently prevails in the community will be maintained indefinitely. Under this assumption, an investment is interpreted by households as creating a permanent increase in public good levels and we are back in a world where the standard logic applies. However, justifying what is quintessentially an economic approach to policy evaluation by jettisoning a standard assumption of economic theory strikes us as unsatisfactory.

On balance, we see the findings of this paper as undermining the case for using the housing price test. Nonetheless, the basic idea of using the housing price response to investments to infer something about the efficiency of durable public good provision may yet hold promise. It is just that a more structural approach may be necessary to exploit this connection. Ultimately, inefficient provision must be driven by a wedge between the objectives of policy-makers and residents. As illustrated in the political economy model of this paper, different wedges will have different implications for the behavior of investment and housing prices. If so, information about wedges and thus efficiency may be recovered from investment and housing price dynamics.

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8 Appendix A: Proofs

8.1 Proof of Proposition 1

Consider a period in which the community's level of public good is g and its tax obligation is T. Recall that the supply of housing is fixed at H. Potential residents just differ in their preference for living in the community θ. Clearly, those with higher θ will have a greater willingness to pay to live in the community. Thus, in equilibrium, the fraction H of potential residents with the highest preference parameters will live in the community. Since

$$\frac{\overline{\theta} - \overline{\theta}\,(1-H)}{\overline{\theta}} = H,$$

the marginal resident will have preference parameter $\overline{\theta}\,(1-H)$. This will be the case in each and every period irrespective of the community's level of public good and tax obligation. It follows that, in equilibrium, potential residents with types $\theta \in [0, \overline{\theta}\,(1-H))$ never reside in the community. For these types, therefore,

$$V_\theta(g,T) = \frac{u}{1-\beta},$$

which yields equation (10) of Proposition 1. Types $\theta \in [\overline{\theta}\,(1-H)), \overline{\theta}]$, on the other hand, will reside in the community as long as they remain in the pool of potential residents. For these types, therefore, irrespective of g and T

$$V_\theta(g,T) = \theta + B(g) - T/H - P(g,T) + \beta E P(g',T') + \beta[\mu E V_\theta(g',T') + (1-\mu)\frac{u}{1-\beta}].$$

We now show that the value functions of the resident households can be written as equation (9) of Proposition 1. Let future periods be indexed by $t = 1, ..., \infty$ and let (g_t, T_t) denote the public good level and tax obligation in period $t = 1, ..., \infty$. If $\theta \in [\overline{\theta}\,(1-H)\,, \overline{\theta}]$, we know that

$$V_\theta(g,T) = \theta + \beta(1-\mu)\frac{u}{1-\beta} + B(g) - T/H - P(g,T) + \beta E\left[P(g_1,T_1) + \mu V_\theta(g_1,T_1)\right], \quad (36)$$

where expectations are taken over the possible values of (g_1, T_1); that is, $((1-\delta)g + I(g,T), cI(g,T))$ and $((1-\delta)g, 0)$. But, since the household will reside in the community in period 1 if it remains in the pool, we also know that

$$\begin{aligned}\beta\left[P(g_1,T_1) + \mu V_\theta(g_1,T_1)\right] \quad = \quad & \beta(1-\mu)P(g_1,T_1) \\ & +\beta\mu\left[\theta + \beta(1-\mu)\frac{u}{1-\beta} + B(g_1) - T_1/H + \beta E\left\{P(g_2,T_2) + \mu V_\theta(g_2,T_2)\right\}\right].\end{aligned}$$

Moreover, period 1's housing price $P(g_1, T_1)$ satisfies the equilibrium condition

$$1 - \frac{\underline{u} - (B(g_1) - T_1/H - P(g_1, T_1) + \beta EP(g_2, T_2))}{\overline{\theta}} = H,$$

which implies that

$$P(g_1, T_1) = \overline{\theta}(1 - H) - \underline{u} + B(g_1) - T_1/H + \beta EP(g_2, T_2).$$

Substituting this into the above expression, we can write

$$\begin{aligned}
\beta \left[P(g_1, T_1) + \mu V_\theta(g_1, T_1)\right] &= \beta(1 - \mu) \left[\overline{\theta}(1 - H) - \underline{u} + B(g_1) - T_1/H + \beta EP(g_2, T_2)\right] \\
&\quad + \beta\mu \left[\theta + \beta(1 - \mu)\frac{u}{1 - \beta} + B(g_1) - T_1/H + \beta E\left\{P(g_2, T_2) + \mu V_\theta(g_2, T_2)\right\}\right] \\
&= \kappa_1(\theta) + \beta\left[B(g_1) - T_1/H\right] + \beta^2 E\left[P(g_2, T_2) + \mu^2 V_\theta(g_2, T_2)\right],
\end{aligned}$$

where

$$\kappa_1(\theta) = \beta\left\{(1 - \mu)\left[\overline{\theta}(1 - H) - \underline{u}\right] + \mu\theta + \beta\mu(1 - \mu)\frac{\overline{u}}{1 - \beta}\right\}.$$

Again, since the household will reside in the community in period 1 if it remains in the pool, we also know that

$$\begin{aligned}
\beta^2 \left[P(g_2, T_2) + \mu^2 V_\theta(g_2, T_2)\right] &= \beta^2(1 - \mu^2)P(g_2, T_2) \\
&\quad + \beta^2\mu^2 \left[\theta + \beta(1 - \mu)\frac{u}{1 - \beta} + B(g_2) - T_2/H + \beta E\left\{P(g_3, T_3) + \mu V_\theta(g_3, T_3)\right\}\right].
\end{aligned}$$

Equilibrium in the housing market implies that

$$P(g_2, T_2) = \overline{\theta}(1 - H) - \underline{u} + B(g_2) - T_2/H + \beta EP(g_3, T_3).$$

Substituting this in, we can write

$$\begin{aligned}
\beta^2 \left[P(g_2, T_2) + \mu^2 V_\theta(g_2, T_2)\right] &= \beta^2(1 - \mu^2)\left(\overline{\theta}(1 - H) - \underline{u} + B(g_2) - T_2/H + \beta EP(g_3, T_3)\right) \\
&\quad + \beta^2\mu^2 \left[\theta + \beta(1 - \mu)\frac{u}{1 - \beta} + B(g_2) - T_2/H + \beta E\left\{P(g_3, T_3) + \mu V_\theta(g_3, T_3)\right\}\right] \\
&= \kappa_2(\theta) + \beta^2\left[B(g_2) - T_2/H\right] + \beta^3 E\left[P(g_3, T_3) + \mu^3 V_\theta(g_3, T_3)\right]
\end{aligned}$$

where

$$\kappa_2(\theta) = \beta^2\left\{(1 - \mu^2)\left[\overline{\theta}(1 - H) - \underline{u}\right] + \mu^2\left[\theta + \beta(1 - \mu)\frac{u}{1 - \beta}\right]\right\}.$$

By similar logic, for all periods $t \geq 3$, we have that

$$\beta^t\left[P(g_t, T_t) + \mu^t V_\theta(g_t, T_t)\right] = \kappa_t(\theta) + \beta^t\left[B(g_t) - T_t/H\right] + \beta^{t+1}E\left[P(g_{t+1}, T_{t+1}) + \mu^{t+1}V_\theta(g_{t+1}, T_{t+1})\right]$$

where

$$\kappa_t(\theta) = \beta^t \left\{ (1-\mu^t)\left[\overline{\theta}(1-H) - \underline{u}\right] + \mu^t \left[\theta + \beta(1-\mu)\frac{\underline{u}}{1-\beta}\right]\right\}.$$

Successively substituting these expressions into (36), reveals that

$$\begin{aligned} V_\theta(g,T) &= \theta + \beta(1-\mu)\frac{\underline{u}}{1-\beta} + B(g) - T/H - P(g,T) + \sum_{t=1}^{\infty} E\left\{\kappa_t(\theta) + \beta^t\left(B(g_t) - T_t/H\right)\right\} \\ &= \sum_{t=0}^{\infty} \kappa_t(\theta) + S(g,T) - P(g,T). \end{aligned}$$

Letting $\kappa(\theta) = \sum_{t=0}^{\infty} \kappa_t(\theta)$ yields equation (9).

It remains to show that equation (11) is satisfied. In equilibrium, it must be the case that the marginal household, which is the household with preference $\overline{\theta}\,(1-H)$, is just indifferent between residing in the community or not. Thus, it must be the case that

$$\kappa(\overline{\theta}(1-H)) + S(g,T) - P(g,T) = \frac{\overline{u}}{1-\beta}.$$

This implies that

$$P(g,T) = \kappa(\overline{\theta}(1-H)) - \frac{\overline{u}}{1-\beta} + S(g,T).$$

Letting

$$K = \kappa(\overline{\theta}(1-H)) - \frac{\overline{u}}{1-\beta},$$

yields equation (11). ■

8.2 Proof of Proposition 2

Letting g' denote next period's public good level (i.e., $g' = (1-\delta)g + I$), we have that

$$S^o(g,T) = \max_{g' \geq (1-\delta)g} B(g) - T/H + \beta S^o(g', c(g' - (1-\delta)g)). \tag{37}$$

Moreover, letting $g'(g)$ denote the optimal policy function for (37), we have that $I^o(g)$ is equal to $g'(g) - (1-\delta)g$. (Note that the linearity of the objective function in T means that the optimal policy will only depend on g.) Now note from (37) that

$$\frac{\partial S^o(g,T)}{\partial g} = B'(g) - \beta(1-\delta)c\frac{\partial S^o(g', c(g' - (1-\delta)g))}{\partial T}, \tag{38}$$

and that

$$\frac{\partial S^o(g,T)}{\partial T} = -1/H. \tag{39}$$

The first order condition for the optimal policy g' is that

$$\frac{\partial S^o(g', c(g' - (1-\delta)g))}{\partial g} + c\frac{\partial S^o(g', c(g' - (1-\delta)g))}{\partial T} \leq 0 \ (= \text{ if } g' > (1-\delta)g).$$

Using (38) and (39), this can be rewritten as

$$B'(g') - \beta(1-\delta)c/H \leq c/H \ (= \text{ if } g' > (1-\delta)g).$$

Thus, if $g' > (1-\delta)g$

$$HB'(g') = c\left(1 - \beta(1-\delta)\right).$$

Letting g^o satisfy the dynamic Samuelson Rule $HB'(g^o) = c\left[1 - \beta(1-\delta)\right]$, we conclude that the optimal policy function is

$$g'(g) = \begin{cases} g^o \ \text{ if } g \leq g^o/(1-\delta) \\ (1-\delta)g \ \text{ if } g > g^o/(1-\delta) \end{cases}.$$

Since $I^o(g)$ is equal to $g'(g) - (1-\delta)g$, Proposition 2 follows. ■

9 Appendix B: Solution algorithm for the model with a budget-maximizing bureaucrat

In this section, we describe the algorithm we use to solve for the equilibrium investment path in the model with a budget-maximizing bureaucrat. We first re-formulate the definition of an equilibrium in a more tractable way. Then we describe the computational procedure for solving for the equilibrium.

9.1 Re-formulating equilibrium

Recall the public good surplus function $S(g, T)$ defined in (8) where

$$S(g,T) = B(g) - T/H + \beta\left[\pi(g)S((1-\delta)g + I(g), cI(g)) + (1-\pi(g))S((1-\delta)g, 0)\right].$$

Note that the variable T, which represents the tax obligation created by past investment, only affects contemporaneous surplus and does not enter the expected surplus in the future. Therefore, we can separate this term from the rest of the terms in the surplus function by defining a new function $\widetilde{S}(g)$ where

$$\widetilde{S}(g) = S(g,T) + T/H. \tag{40}$$

Substituting (40) into (8) and using the notation $g'(g) = (1-\delta)g + I(g)$ to denote next period's equilibrium public good level, we obtain the following recursive representation of $\widetilde{S}(g)$

$$\widetilde{S}(g) = B(g) + \beta \left[\pi(g) \left(\widetilde{S}(g'(g)) - \frac{c[g'(g) - (1-\delta)g]}{H} \right) + (1-\pi(g))\widetilde{S}((1-\delta)g) \right]. \tag{41}$$

The probability of investment approval defined in equation (30) can hence be re-written as

$$\widetilde{\varphi}(g, g') = \Phi \left(\widetilde{S}(g') - \frac{c[g' - (1-\delta)g]}{H} - \widetilde{S}((1-\delta)g) \right). \tag{42}$$

Based on (41) and (42), the bureaucrat's maximization problem (31) can be rewritten as

$$g'(g) = \arg\max \left\{ \begin{array}{c} u(g) + \beta \left[\widetilde{\varphi}(g, g')U(g') + (1 - \widetilde{\varphi}(g, g'))\, U((1-\delta)g) \right] \\ s.t. \; g' \geq (1-\delta)g \end{array} \right\}. \tag{43}$$

The probability the proposal will be implemented, $\pi(g)$, will then satisfy the equality $\pi(g) = \widetilde{\varphi}(g, g'(g))$ and the bureaucrat's value function will satisfy the functional equation

$$U(g) = u(g) + \pi(g)U(g'(g)) + (1-\pi(g))\, U((1-\delta)g). \tag{44}$$

We can now define an equilibrium to be a pair of functions $g'(g)$ and $\pi(g)$ such that, when the public good surplus function $\widetilde{S}(g)$ is defined by (41), the probability of approval $\widetilde{\varphi}(g, g')$ is defined by (42), and the bureaucrat's value function is defined by (44), the function $g'(g)$ solves problem (43) and the implementation probability function $\pi(g)$ equals $\widetilde{\varphi}(g, g'(g))$. The associated equilibrium investment function $I(g)$ can be recovered from the equation $I(g) = g'(g) - (1-\delta)g$.

9.2 Numerical procedure

We solve for the equilibrium by iteration over the value functions $U(g)$ and $\widetilde{S}(g)$. The exact steps are as follows:

Step 1. Construct an equally spaced grid $\mathcal{G}$ over the interval of possible public good levels $[g_{min}, g_{max}]$.[20]

Step 2. Given an initial guess for the public good surplus function $\widetilde{S}_0(g)$, compute the associated probability of investment approval function $\widetilde{\varphi}_0(g, g')$ defined in (42) for each pair $(g, g') \in \mathcal{G} \times \mathcal{G}$. When computing $\widetilde{S}_0((1-\delta)g)$, if $(1-\delta)g$ is outside the $\mathcal{G}$ grid, use piecewise polynomial

[20] We set g_{max} sufficiently large so that $g'(g)$ never exceeds it. We set g_{min} equal to 0 for Example 1 and Example 3 where $\gamma = 0$. For Example 2 where $\gamma = 5$, the bureaucrat's utility $u(g)$ is not well-defined at 0. Therefore we set $g_{min} = 0.5$ for this example.

interpolation.

Step 3. For each g value in grid $\mathcal{G}$, using $\tilde{\phi}_0(g, g')$ and an initial guess for the bureaucrat's value function $U_0(g)$, compute the bureaucrat's objective function in problem (43) at each $g' \in \mathcal{G}$. As in Step 2, if $(1-\delta)g$ is outside the $\mathcal{G}$ grid, use piecewise polynomial interpolation to obtain $U_0((1-\delta)g)$. Then choose the $g' \in \mathcal{G}$ which maximizes the bureaucrat's objective subject to satisfying the constraint that $g' \geq (1-\delta)g$. This generates the equilibrium policy function $g_0'(g)$ associated with the initial guess $\left(U_0(g), \widetilde{S}_0(g)\right)$.

Step 4. Using $g_0'(g)$, $\left(U_0(g), \widetilde{S}_0(g)\right)$, $\tilde{\phi}_0(g, g')$, along with equations (41), (42), and (44), form a new guess for the value functions $\left(U_1(g), \widetilde{S}_1(g)\right)$. If the distance between $\left(U_1(g), \widetilde{S}_1(g)\right)$ and $\left(U_0(g), \widetilde{S}_0(g)\right)$ is less than an exogenously specified tolerance, then the equilibrium value functions are found and the associated investment path can be obtained from the functions $g_0'(g)$ and $\tilde{\phi}_0(g, g_0'(g))$. Otherwise we replace the initial guess with $\left(U_1(g), \widetilde{S}_1(g)\right)$ and repeat Steps 2 through 4.