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Optimal Fiscal Limits*

Abstract

This paper studies the optimal design of fiscal limits in the context of a simple political economy model. The model features a single politician and a representative voter. The politician is responsible for choosing the level of public spending for the voter but may be biased in favor of spending. The voter sets a spending limit and requires that the politician have voter approval to exceed it. This limit must be set before the voter's preferences for public spending are fully known. The paper first solves for the optimal limit and explains how it depends upon the degree of politician bias and the nature of the uncertainty concerning the voter's preferred spending level. A dynamic version of the model is then analyzed and policies which limit the rate of growth of spending are shown to dominate those that cap spending to be below some fixed fraction of community income.

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1 Introduction

Tax and spending limits are a common feature of the state and local government fiscal landscape in the US. At the state level, Rose (2010) and Waisanen (2010) report that thirty states operate under a tax or spending limitation.[1] In some states these limits are constitutional and in others they are statutory. They have been implemented both by state legislative bodies and directly by citizens through the initiative process. At the local level, Mullins (2010) reports that all but three states impose some form of constitutional or statutory statewide limitation on the fiscal behavior of their local governments.[2] Moreover, self-imposed limits are quite prevalent at the local level as recently documented by Brooks, Halberstam, and Phillips (2013).[3]

Limits come in many forms and apply to a variety of different fiscal variables. With respect to taxes, there are limits on tax *rates*. In particular, limits on property tax rates are very common at the local level. There are also limits on the total amount of tax that can be raised, so called *tax levy* limits. These can apply to revenue raised from a specific tax or to total tax revenue. With respect to spending, there are limits on the total amount of spending that the government can do. All limits have override provisions which specify when limits may be violated. Typically, violation requires either a super-majority vote of the governing legislative body or direct approval of a majority or super-majority of citizens in a referendum.

Limits typically govern fiscal policy for a period of time, rather than being reoptimized at the beginning of each fiscal year. Moreover, they differ in how they are structured over time. One common form of spending limit, for example, simply caps spending each year to be below some fixed fraction of the community's aggregate income.[4] Another common form requires that spending may not increase annually by more than some percentage. This percentage typically depends on the growth of community aggregate income, population, and/or inflation.[5] These

[1] This is a conservative estimate, since Mullins (2010) reports that thirty five states have limitations. Eighteen states have revenue limits, twenty seven have spending limits, and nine have provisions limiting both revenue and spending.

[2] The three exceptions are Connecticut, New Hampshire, and Vermont. The most common type of state-imposed limit on local governments is a property tax rate limitation (thirty three states). Tax levy limits are also common (thirty states). Nine states limit spending growth in their local governments and two limit revenue increases. States also regulate what their local governments can tax. See Mullin (2010) for more details.

[3] These researchers find that one in eight of the municipalities in their large-scale survey have self-imposed limits (as distinct from state-imposed limits).

[4] According to Waisanen (2010), such a spending limit exists in Arizona, Idaho, Missouri, and North Carolina.

[5] According to Waisanen (2010), such a spending limit exists in Connecticut, Hawaii, Maine, New Jersey,

two forms of spending limit are equivalent only when spending is always equal to the limit. This will not be the case when, for example, an override is approved. Similarly, tax rate limits differ in what happens when the limit is overriden and a higher rate is approved. In some cases, the new higher rate becomes the new limit. In others, the limit reverts back to its original level the next period. In still others, politicians are required to propose for how many periods they want the higher rate.[6]

The purpose of this paper is to consider the optimal design of fiscal limits. Thus, the paper seeks to develop principles that might guide citizens in the setting of such limits. Questions of interest include how stringent should limits be and what does the optimal stringency depend on? What is the role of the override provision? How should limits be structured over time?

The paper begins by developing a simple model in which to consider the problem. The agents in the model are a politician who is in charge of selecting the level of spending on public goods and services for a community and a representative voter who benefits from the public spending but has to pay taxes to finance it. The voter's preferred spending level is some fraction of the community's income, but this fraction is ex ante uncertain. The politician may be biased in favor of spending in the sense that he prefers to see a higher fraction of community income spent on public goods and services than the voter. The voter sets a spending limit before his preferred spending level is known. Following common practice, the limit is in the form of a maximal fraction of community income that can be devoted to public spending. Moreover, the limit has an override provision that allows the politician to violate it with the voter's approval.

The paper solves for the optimal limit and explains how it depends upon the extent of politician bias and the nature of the uncertainty concerning the voter's preferred spending level. When the politician's bias exceeds a threshold, the optimal limit equals the *expected* fraction of community income the voter would like devoted to public spending. Below this threshold, the limit exceeds this level. For some distributions of the voter's preferred spending level, the optimal limit is more permissive the lower is the politician's bias and, as this bias goes to zero, converges to the maximum fraction of income the voter could possibly want devoted to public spending. Surprisingly, however, for other distributions, the limit becomes more stringent as the politician's bias decreases. Examples suggest that greater uncertainty in the voter's preferred spending level

Oklahoma, South Carolina, Utah, and Washington.

[6] For examples of all three of these systems in practice see Chapter 2 of Hamilton and Cohen (1974) which inventories the rich variety of school finance systems across the states.

results in a more permissive limit.

The paper then extends the model to a dynamic setting to analyze how limits should be structured over time. The extension assumes that the fraction of the community's income the voter would like to see devoted to public spending evolves according to a stochastic process. Shocks are persistent in the sense that a positive demand shock in the current period implies an increase in demand for public spending in the future. This creates a dynamic linkage across periods. A different politician is in office each period, so that politicians behave myopically. The analysis first assumes that a sequence of limits must be chosen at the beginning of the first period. The optimal sequence involves capping spending in each period to be below a constant fraction of community income. Under our assumptions on the magnitude of bias, this fraction is the expected fraction of community income the voter would like to see devoted to public spending conditional on the information available at the beginning of the first period. However, this optimal sequence is shown to be dominated by a system of spending-contingent limits that, after capping spending in the first period, limit the growth of future spending to be less than the growth of community income. Thus, the analysis suggests that systems that limit the growth of spending dominate those that simply cap spending.

The organization of the remainder of the paper is as follows. Section 2 discusses related literature. Section 3 outlines the basic model and analyzes the optimal limit in the static case. Section 4 extends the model to incorporate dynamics and studies the optimal way to structure spending limits over time. Section 5 concludes with a brief summary of the findings and suggestions for further research.

2 Related literature

There is a large literature on fiscal limits.[7] This literature focuses on three main tasks. The first is documenting the types of limits faced by state and local governments and describing when and how they were introduced (see, for example, Mullins 2010 and Waisanen 2010). This is difficult and time consuming because there is a great deal of variation across states and localities and a considerable amount of change over time. The second task is understanding how limits impact

[7] Selective reviews are provided by Krol (2007), Mullins and Wallin (2004), and Rose (2010).

the fiscal variables they seek to regulate and other related public policies.[8] This is challenging because of the problem of identifying the effect of limits. The third task is understanding what citizens think about existing limits and why they were introduced (see, for example, Citrin 1979, Courant, Gramlich and Rubinfeld 1985, Cutler, Elmendorf, and Zeckhauser 1999, and Ladd and Wilson 1982).

The normative question of whether limits enhance citizens' welfare and, if so, what should be limited and how should limits be designed has attracted less attention in the literature. Brennan and Buchanan (1979) provide an early and wide ranging normative discussion of tax limits. They study the issue in the context of a model in which a Leviathan government wastes a fixed fraction of any revenues raised for public good provision. This Leviathan government would like to maximize revenues raised. Brennan and Buchanan discuss a number of different limits: tax rate limits, tax levy limits, and tax base limits.[9] They consider tax levy limits that require revenue to be less than some fraction of community income and argue that assessing the appropriate fraction will be too complicated for average citizens. Our analysis seeks to provide guidance on exactly this question as it applies in the spending context. They also question whether such limits can be effective in restraining government, arguing that the footprint of government in the economy does not equate to the tax revenue it raises. In particular, they point out that government can intervene with non-tax methods such as mandates and regulations. This concern is abstracted from in this paper.

Besley and Smart (2007) study the operation of a tax revenue limit in the context of a two period political agency model. The politician holding office in each period chooses taxes and provides a public good, the cost of which is uncertain. Politicians can be good or bad. Good politicians maximize voters' welfare in an unstrategic way. Bad politicians are strategic and get utility from holding office and diverting tax revenues to their own consumption. The important point that Besley and Smart make is that a revenue limit in the first period not only limits the choices of the incumbent politician but also impacts how much voters learn about the incumbent.

[8] Papers in this branch of the literature include Bradbury, Mayer, and Case (2001), Brooks, Halberstam, and Phillips (2013), Dye and McGuire (1997), Figlio and Rueben (2001), Knight (2000), Poterba (1994), and Poterba and Rueben (1995, 2001). After surveying the literature, Rose (2010) concludes "tax and expenditure limits appear to be modestly effective in slowing the growth of government, particularly at the municipal level, although the evidence is somewhat mixed".

[9] Base limits correspond to restrictions on what the government may tax. For example, local governments in many states are not allowed to tax income.

In particular, a revenue limit might induce a pooling equilibrium between good and bad politicians in the first period, which leads to worse selection in the second period. This impact must be taken into account in a full welfare analysis of limits. This point is abstracted from in our dynamic analysis which assumes that politicians hold office for one period only.

As a prelude to their empirical work, Brooks, Halberstam, and Phillips (2013) provide a theoretical analysis that is closer to this paper. They develop a framework for understanding limits building on a model of local government elections presented in Coate and Knight (2011). There are two groups of citizens with low and high preferences for public goods. The level of public good is chosen by an elected politician. Politicians are citizens and choose their preferred public good level if elected. However, citizens cannot observe candidates' preferences. A limit is implemented when the majority have low preferences and is intended to constrain high spending politicians. The cost of the public good is uncertain which makes the choice of limit non-trivial. The optimal limit is shown to be more permissive the higher the probability the elected politician is a low type. However, the analysis assumes there is no override. While it does not explicitly incorporate elections, our model is consistent with that of Brooks, Halberstam, and Phillips. Nonetheless, our analysis of optimal limits differs from theirs because we incorporate the reality that limits can be overriden. This changes the calculus of the optimal limit.

The model presented here is also related to the well-known agenda setter model of Romer-Rosenthal (Romer and Rosenthal 1978 and 1979). The agenda setter model considers the interaction between a politician and a representative voter. The voter's utility depends on the level of public spending and the politician is responsible for choosing the level of this spending. The politician is not only biased in favor of spending in the sense that he always prefers a higher level than the voter, he is in fact a budget maximizer. The politician's proposed spending level must be approved by the voter and, if it is not, then an exogenous reversion level is implemented. In equilibrium, the politician proposes a spending level which leaves the voter indifferent between the proposal and the reversion level. The proposed spending level exceeds the reversion level whenever the latter falls below the voter's preferred spending level. In this paper, the choice of the limit can be thought of as endogenizing the reversion level. Moreover, the fact that the limit must be chosen before the voter's preferences are fully known makes the choice of limit interesting even in the case in which the politician is a budget maximizer and thus heavily biased.

More generally, the paper contributes to a broader literature on fiscal constitutions. A fiscal constitution is a set of rules and procedures that govern the determination of fiscal policies (see, for example, Brennan and Buchanan 1980). It is distinct from a political constitution which sets up the architecture of government and the rules by which policy-makers are selected. The fiscal constitution literature seeks to understand the effectiveness of various rules and procedures in generating good fiscal policies for citizens. In addition to tax and spending limits, it studies balanced budget rules, budgetary procedures, debt limits, and rainy day funds. Rose (2010) provides a useful review of this literature.

Finally, the paper is related to the contract theory literature on the *delegation problem* (see, for example, Alonso and Matouschek 2008, Amador and Bagwell 2013, Holmstrom 1977, 1984, and Melumad and Shibano 1991). In this problem, a principal interacts with an informed but biased agent and contingent transfers between the principal and agent are ruled out. The nature of the interaction is that the principal chooses a set of permissible actions for the agent and, given his private information, the agent chooses his preferred action from this set. The question of interest is what is the optimal set of permissible actions from the principal's perspective? This general theoretical problem has many interesting applications, including several in the field of political economy.[10]

At first glance, the delegation problem seems to map cleanly into our setting: the principal is the representative voter, the agent is the politician, and the permissible actions are the spending levels lower than the limit. Moreover, information is asymmetric in the sense that at the time the voter has to choose the limit he does not have perfect information but knows that the politician will be fully informed when choosing the spending level. The choice of the limit determines the set of permissible actions and hence the question of the optimal limit is relevant to the problem of determining the optimal set of permissible actions.[11] Nonetheless, there are important differences between the problem considered here and the delegation problem. In our problem, there is an override provision so that the politician can choose a policy outside the set of permissible actions with the voter's approval.[12] Moreover, if the politician does decide to take advantage of the

[10] One application is the delegation of policy-making from elected politicians to bureaucrats (see, for example, Epstein and O'Halloran 1994 and Huber and Shipan 2006). Another is to the delegation of policy-making from legislatures to standing committees (see, for example, Gilligan and Krehbiel 1987, 1989 and Krishna and Morgan 2001).

[11] Of course, simply imposing a limit may not be the optimal way of constructing the set of permissible actions.

[12] In the delegation problem setting, Mylovanov (2008) shows that the principal can implement an optimal outcome through *veto-based delegation* with an appropriately chosen default policy. In this implementation, the

override provision, the voter is fully informed when deciding whether to approve the politician's proposal. Thus, there is no information asymmetry between the voter and politician at this stage of the interaction.[13]

Indeed, the fiscal limit we study better resembles an *incomplete contract* between the representative voter and the politician.[14] It is incomplete because the limit is not conditional on the voter's preferred spending level even though both parties can observe this variable when it is realized. The real world justification for this is that voters' preferred spending levels depend on too many factors to be coded into a limit formula. Like an incomplete contract, the fiscal limit also specifies a renegotiation procedure through the override process. The politician has the property right to choose the spending proposal and voters have the right to veto it if it exceeds the limit. In contrast to the incomplete contracting literature, however, our analysis does not seek to explain why the incomplete contract defined by the fiscal limit is optimal. It simply focuses on the narrow, but practically relevant, question of what determines the optimal limit. Thus, we are optimizing within this particular class of incomplete contracts.

3 Static analysis

3.1 The static model

A politician is in charge of selecting a level of spending on public goods and services for a community. A representative voter benefits from public spending but has to pay taxes to finance it. The voter desires that a certain fraction of the community's income be devoted to public spending, but this preferred fraction is ex ante uncertain. The politician may share the voter's preferences over spending or may prefer a higher level. The voter is aware of the politician's potential spending

agent proposes a policy and then the principal approves it or not. If he does not approve it, the default policy is implemented. However, this differs from the institution considered in this paper which requires that the politician obtain the voter's approval only if he exceeds the limit. Moreover, in Mylovanov's scheme the voter is not fully informed when he is voting on the agent's proposal. Rather he makes inferences about what must be true from the politician's proposal.

[13] This may appear similar to the set-up of Epstein and O'Halloran (1994). In their well-known model, a politician must decide how much discretion to provide to a bureaucrat. The bureaucrat makes a policy proposal within an interval of permissible proposals set by the politician. After the bureaucrat has made his proposal, the politician may veto it. If he does so, some default policy is implemented. As in our model, at the time of the veto decision, the politician is fully informed. However, this model differs from ours in that i) the bureaucrat's proposal is always subject to the politician's veto, and ii) the bureaucrat cannot choose from outside the interval of permissible policies. In our model, the voter (who corresponds to the politician in their model) only votes if the politician (who corrsponds to the bureaucrat) proposes something outside the permissible set.

[14] For textbook treatments of incomplete contracts see, for example, Bolton and Dewatripont (2004) and Salanie (1997).

bias and, before he knows his preferred spending level, imposes a spending limit on the politician. This limit is in the form of a maximal fraction of community income that can be devoted to public spending. The limit comes with an override provision that allows the politician to violate it with the voter's approval.

The level of spending is denoted s and the community's income by y. The voter's preferred spending level is σy. The voter has distance policy preferences $-|s-\sigma y|$ so that his utility declines linearly and symmetrically as the spending level diverges in either direction from his ideal.[15] The voter's preferred fraction of community income to devote to public spending σ (hereafter *preferred fraction*) is the realization of a random variable with range $[\underline{\sigma}, \overline{\sigma}]$ and cumulative distribution function $H(\sigma)$. The associated density function, $h(\sigma)$, is assumed to be symmetric around the mean $\sigma_m = (\underline{\sigma}+\overline{\sigma})/2$.[16] In addition, the density is continuous and non-decreasing on $[\underline{\sigma}, \sigma_m]$. These assumptions imply that the cumulative distribution function is convex on the interval $[\underline{\sigma}, \sigma_m]$ and concave on the interval $[\sigma_m, \overline{\sigma}]$.

With probability π the politician is biased in favor of higher spending and with probability $1-\pi$ he shares the voter's preferences. When biased, the politician has preferences $-|s-(1+b)\sigma y|$ so that his preferred spending level is $(1+b)\sigma y$. The parameter b therefore measures the magnitude of the biased politician's spending bias.

The spending limit is denoted l. The limit is in the form of a maximal fraction of the community's income that can be devoted to public spending, so it prevents the politician from implementing a spending level in excess of ly without the voter's approval. Without loss of generality, the limit l is assumed to belong to the interval $[\underline{\sigma}, \overline{\sigma}]$.

The timing of the interaction between the voter and the politician is as follows. First, knowing y, H, π, and b, the voter selects a limit l from the interval $[\underline{\sigma}, \overline{\sigma}]$. Second, nature selects the voter's preferred fraction σ which is observed by both players. Third, the politician proposes a spending level s. If the proposal does not exceed the maximal permitted spending level ly it is implemented. Fourth, if the proposed spending level violates the limit, an election is held and the voter votes in favor or against the proposal. If he votes in favor, the proposal is implemented. Fifth, if the

[15] While not without their drawbacks (see, for example, Milyo 2000), distance preferences are very widely used in the political economy literature. They are both analytically tractable and simple to understand. The particular form used here assumes a linear loss of utility for the voter as spending diverges from his ideal. This is distinct from a quadratic loss which is also commonly assumed.

[16] This means that for any σ below the mean σ_m, $h(\sigma)$ is equal to $h(2\sigma_m - \sigma)$.

voter votes against the proposal, the politician chooses another spending level s' which respects the limit and this is implemented.

3.2 Discussion of the model

The model raises a number of obvious questions which it is useful to briefly discuss. One question is why might the politician be biased? In particular, why cannot voters always elect a candidate who shares their spending preferences? Of course, to the extent that elections select in the right candidates, then limits would seem unnecessary. Thus, the prevalence of limits at the state and local government level suggests that candidate elections cannot be working perfectly in these environments.[17] To explain this imperfect functioning, it is common to point out that elections at this level of government are small scale affairs and that, as a consequence, voters are not well informed about candidates' policy preferences. Moreover, the rewards to holding office are not large enough to provide incentives for elected candidates to diverge from their preferences to increase their chances of re-election. But all this only means that, when elected, candidates will likely follow their policy preferences and that these preferences may not be congruent with those of the median voter. It does not explain a particular direction of bias. For this, there are (at least) two possible explanations. First, it may be that interest groups operate to put pressure on elected leaders to increase spending above the level preferred by voters. Many stakeholders stand to benefit from public spending. These include public employees, public contractors, and recipients of government grants. By the usual logic of concentrated benefits versus diffuse costs, these stakeholders may form groups to influence politicians. In such environments, politicians may act as if they prefer higher spending even if, as citizens, they share the general voter's preferences (see, for example, Grossman and Helpman 1994 and Besley and Coate 2001). The second explanation is selection. For certain local government offices, it is reasonable to believe that the people most likely to run are those who care intensely about the policies the office controls and thereby have higher preferences for spending on these policies. Good examples might be school board or town and city council.

A related question is how does the voter know the politician's exact bias (assuming he is biased)? Would it not be more realistic to just assume the voter was uncertain of the degree of

[17] This said, it is possible to generate an explanation for the limits states impose on their local governments while assuming that the median voter theorem applies at both the state and local level. See Calabrese and Epple (2011) and Vigdor (2004).

the politician's bias, allowing in effect a continuous distribution of bias rather than a two point distribution? The answer to this question is obviously yes. A known level of bias is assumed purely for reasons of tractability.[18] The optimal limit design problem is quite complicated with a known level of politician bias and it makes sense to understand this problem prior to introducing a more general type of uncertainty.

Another question is why is the voter uncertain about the fraction of community income he would like to see devoted to public spending? If there were no such uncertainty, the voter should just impose a limit equal to his preferred fraction and that would be the end of the analysis. In particular, there would be no role for overrides. The ubiquity of override provisions suggests that in the real world there must be uncertainty. Moreover, such uncertainty seems intuitively plausible. The type of uncertainty will depend on the nature of the policy the politician is controlling. If the policy is road maintenance (snow plowing, pothole repair, etc), then uncertainty would be created by weather, the prices of inputs like road salt, tarmac, etc. If the policy is police protection then uncertainty would be created by the underlying forces generating crime. If the policy is school spending then uncertainty would be created by the prices of school supplies, mandates from higher levels of government, and state and federal financial support.

A further question is why does the model assume that the voter knows the level of community income? If voters are uncertain about the fraction of community income they would like to see devoted to public spending, it also seems likely they will be uncertain of the level of community income. This is certainly a reasonable point. Fortunately, as we will show below, given the assumed form of preferences and the fact that the limit is expressed in the form of a maximal fraction of community income, uncertainty in the community income level can be introduced without changing the results. Thus, the assumption that the voter knows the level of community income is without loss of generality.

A final question is why the voter cannot simply implement his preferred spending level once uncertainty is resolved? This reflects the assumption that the politician has the property right to choose the spending proposal and citizens only have the right to veto it if it exceeds the limit. The model makes this assumption because it seems an accurate description of reality but does not explain why this arrangement exists. One could certainly imagine alternative arrangements

[18] In the literature on the delegation problem, it is typical to assume a single level of bias (i.e., a one point distribution).

whereby voters could propose alternatives to the politician's proposal. Intuitively, the underlying reasons why we do not observe such arrangements would seem to include i) that only the politician is likely to have the knowledge and expertise to implement a spending plan and ii) that, if citizens could also propose alternatives, it is not clear how to choose between all the alternatives that might be proposed.

3.3 The static limit design problem

We are now ready to consider the problem of choosing a spending limit to maximize the voter's expected welfare. To pose the limit design problem formally, we must understand the policy implications of any given limit l. Let the voter's preferred fraction be σ. There are two possibilities: either the politician is biased or he is unbiased.

Suppose that the politician is biased. Working backwards, consider what policy the politician would choose if he had to satisfy the limit. He will choose a spending level equal to the maximal permitted level ly if this is smaller than his optimal level $(1+b)\sigma y$. Otherwise, he will choose his optimal level. Thus his policy choice will be $\min\{ly, (1+b)\sigma y\}$. The voter will recognize that if he votes down any alternative policy proposed by the biased politician, the spending level implemented will be $\min\{ly, (1+b)\sigma y\}$. If the voter's preferred fraction σ is less than l he will prefer this policy to any higher spending level and there is no point in the politician proposing to violate the limit. In this case, therefore, the implemented spending level will be $\min\{ly, (1+b)\sigma y\}$. If σ exceeds l, the voter will support spending in excess of the limit. The optimal policy proposal for the biased politician solves the problem

$$\begin{gathered} \max_s - |s-(1+b)\sigma y| \\ s.t. - |s-\sigma y| \geq - |ly-\sigma y| \,. \end{gathered} \tag{1}$$

The constraint guarantees that the voter supports the proposal since the politician will choose spending level ly if his proposal is rejected. From the constraint, the maximum spending level the voter will support is $(2\sigma - l)\, y$. The politician will propose this if it is smaller than his preferred level $(1+b)\sigma y$. Otherwise, he will choose his preferred level. In summary, therefore, with a biased politician, the policy implemented under limit l will be $\min\{ly, (1+b)\sigma y\}$ if the voter's preferred fraction σ is less than l and $\min\{(2\sigma - l)\, y, (1+b)\sigma y\}$ otherwise.

If the politician is unbiased, he will choose the spending level σy if the voter's preferred fraction

σ is less than l. Otherwise, he will propose the spending level σy and the voter will approve his proposal.

Putting all this together, it follows that with limit l, the voter's expected welfare will be given by

$$-\pi y \left(\int_{\underline{\sigma}}^{l} \left[\min\{l, (1+b)\sigma\} - \sigma\right] h(\sigma) d\sigma + \int_{l}^{\overline{\sigma}} \left[\min\{2\sigma - l, (1+b)\sigma\} - \sigma\right] h(\sigma) d\sigma \right). \quad (2)$$

The limit design problem is to choose a spending limit from the interval $[\underline{\sigma}, \overline{\sigma}]$ to maximize this function. Since the constraint set is compact and the objective function continuous, the problem has a solution.

There are several observations to make about this problem. First, by inspection, the optimal limit is independent of the probability that the politician is biased π.[19] This reflects the fact that an unbiased politician always chooses the voter's preferred spending level. If the limit is below the voter's preferred fraction, the politician just gets the voter's support to override it. Nonetheless, as we will explain below, the possibility of the politician being unbiased plays a key role in rationalizing the override provision.

Second, the optimal limit is independent of the level of community income y. This means that ex ante uncertainty concerning community income can be incorporated into the model with no change in the analysis. This independence reflects two considerations. One is that the limit is expressed as a maximum fraction of community income. This means that the implied maximum spending level automatically adjusts with the community's income level.[20] The other is that the voter's preferred fraction is assumed to be independent of the community's income level. This means that the voter does not wish to change the proportion of community income spent on public goods and services when this income changes.

Third, removing the multiplicative term πy and rearranging, the limit design problem can be restated as choosing a limit to maximize:

$$V(l) = \int_{\max\{\underline{\sigma}, \frac{l}{1+b}\}}^{l} \left[(1+b)\sigma - l\right] h(\sigma) d\sigma + \int_{l}^{\min_{+}\{\overline{\sigma}, \frac{l}{1-b}\}} \left[l - (1-b)\sigma\right] h(\sigma) d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma) d\sigma, \quad (3)$$

[19] It is worth contrasting this result with that of Brooks, Halberstam, and Phillips (2013). As noted in the introduction, they show that the optimal limit is more permissive the higher the probability the elected politician is a low spending type. A low type politician, in our terms, would be unbiased. Their result differs from ours because they assume the limit cannot be overriden.

[20] Indeed, this feature may explain the popularity of such limits in practice.

where $\min_+\{\overline{\sigma}, l/(1-b)\}$ denotes the smallest positive number of the two.[21] The third term in this expression measures the voter's loss of welfare if there was no limit and the biased politician were to just choose his preferred fraction $(1+b)\sigma$. The first two terms represent the surplus the voter can claw back through the limit. The limit design problem is then to find the limit l that maximizes these first two terms.

3.4 The optimal limit with large politician bias

When the biased politician's bias is large, the limit design problem is straightforward to solve. In particular, suppose that it were the case that b exceeded $(\overline{\sigma} - \underline{\sigma})/\underline{\sigma}$. Then $l/(1+b)$ would be less than $\underline{\sigma}$ for any limit l in the range $[\underline{\sigma}, \overline{\sigma}]$. Moreover, if it were positive, $l/(1-b)$ would exceed $\overline{\sigma}$ for any limit l in the range $[\underline{\sigma}, \overline{\sigma}]$. Accordingly, from (3), the objective function $V(l)$ would be

$$V(l) = \int_{\underline{\sigma}}^{l} [(1+b)\sigma - l]\, h(\sigma)d\sigma + \int_{l}^{\overline{\sigma}} [l - (1-b)\sigma]\, h(\sigma)d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma)d\sigma. \tag{4}$$

Differentiating this expression, we obtain

$$V'(l) = 1 - 2H(l). \tag{5}$$

Recall that H is assumed to have a density that is symmetric around the mean and hence $H(\sigma_m)$ is equal to 1/2. Thus, the voter's welfare is increasing in the limit as long as it is less than σ_m and decreasing thereafter. The optimal limit is therefore equal to σ_m - the *expected* fraction of community income the voter would like to see devoted to public spending.

The intution here is equally transparent. When the biased politician's bias exceeds $(\overline{\sigma} - \underline{\sigma})/\underline{\sigma}$, then, whatever the limit, he will always choose a spending level equal to the maximum allowable level under the limit (ly) when the voter's preferred fraction is less than the limit $(\sigma < l)$. Moreover, when the voter's preferred fraction exceeds the limit $(\sigma > l)$, the biased politician will choose a spending level that provides the voter with exactly the same payoff as he would get from the maximum allowable level under the limit (ly). As a result, the voter's payoff is exactly that which would arise if the policy were just set equal to the maximum level permitted by the limit (ly). The optimal limit is therefore the fraction which, if committed to ex ante, would yield the voter the highest expected payoff. This is the expected preferred fraction σ_m.

[21] If b exceeds 1, $l/(1-b)$ will be negative and hence $\min\{\overline{\sigma}, l/(1-b)\}$ equals $l/(1-b)$ while $\min_+\{\overline{\sigma}, l/(1-b)\}$ equals $\overline{\sigma}$.

In fact, we can weaken the requirement on bias and still keep the conclusion that the limit should equal the expected preferred fraction. As the following proposition shows, it is sufficient that the degree of bias b exceeds $(\overline{\sigma} - \sigma_m)/\underline{\sigma}$.

Proposition 1 *In the static model, if the biased politician's bias b exceeds $(\overline{\sigma} - \sigma_m)/\underline{\sigma}$, the optimal limit is σ_m.*

With a level of bias between $(\overline{\sigma} - \sigma_m)/\underline{\sigma}$ and $(\overline{\sigma} - \underline{\sigma})/\underline{\sigma}$, it remains the case that the voter's payoff from limit σ_m with a biased politician is exactly that which would arise if the spending level were just set equal to $\sigma_m y$ ex ante. However, for a limit l in excess of σ_m, it could be the case that the politician would choose his preferred policy $(1+b)\sigma y$ rather than ly for sufficiently small σ.[22] If so, the payoff from such a limit would strictly exceed that associated with just choosing spending level ly ex ante. Similarly, for a limit l less than σ_m, it could be the case that the politician would choose $(1+b)\sigma y$ rather than $(2\sigma - l)\,y$ for sufficiently large σ. It therefore becomes less obvious that the optimal limit is σ_m because the payoff from alternative limits may have improved. However, the proof of the Proposition shows that, under the conditions on the density function h, σ_m remains optimal.

A graphical interpretation of the result is provided in Figure 1. In each panel, the range of values for the voter's preferred fraction σ is measured on the horizontal axis. The three upward sloping lines are $(1+b)\sigma$, σ, and $(1-b)\sigma$ respectively. The parameters are chosen so that b is exactly equal to $(\overline{\sigma} - \sigma_m)/\underline{\sigma}$. In Panel A, the shaded area represents the surplus that the voter would lose if the biased politician were to choose his preferred fraction $(1+b)\sigma$. This area is the third term in (3). In Panel B, the shaded area represents the surplus generated for the voter by the limit σ_m, which corresponds to the first two terms of (3). Panel C illustrates the surplus generated for the voter by a limit l larger than σ_m. Notice that with this limit, the biased politician chooses his preferred fraction $(1+b)\sigma$ when σ is sufficiently low. The difference in surplus from limit σ_m as opposed to limit l is illustrated in Panel D. In the proof of Proposition 1, this difference is shown to equal the difference between twice the striped area ($\int_{\sigma_m}^{l} [l - \sigma]\, h(\sigma) d\sigma$) less the shaded area ($\int_{\underline{\sigma}}^{\frac{l}{1+b}} [l - (1+b)\sigma]\, h(\sigma) d\sigma$). The assumptions on the distribution function H and bias parameter b imply that this difference is positive.

[22] This requires that $(1+b)\underline{\sigma}$ is less than l.

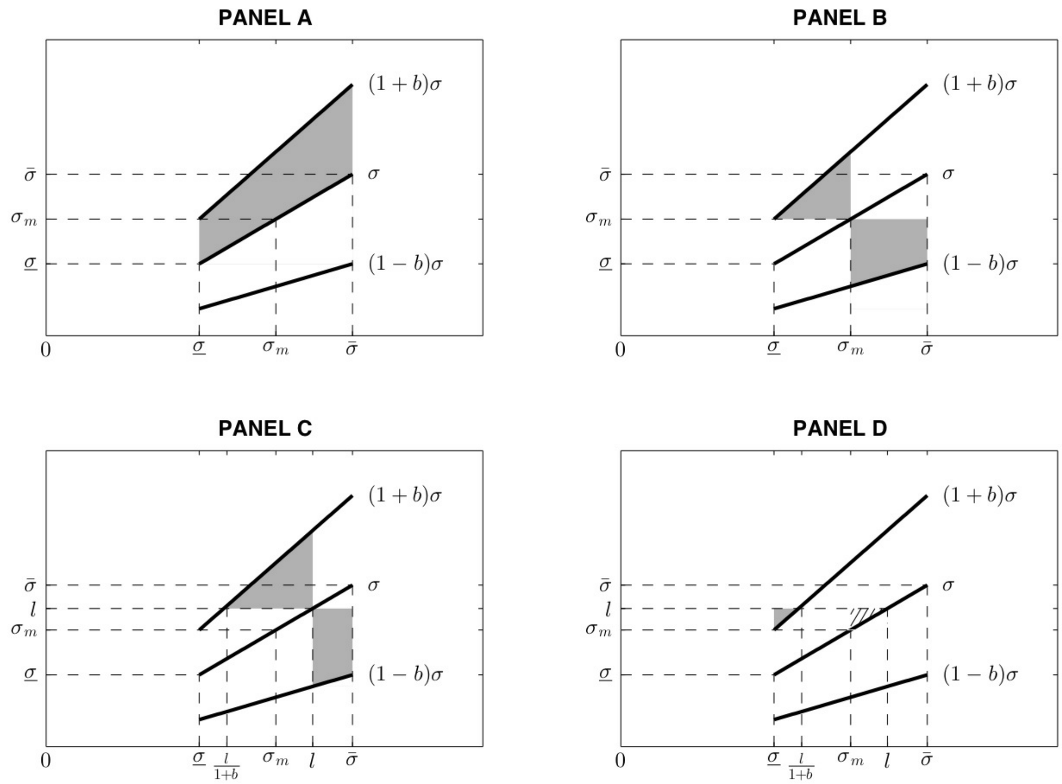

Figure 1: Graphical Interpretation of Proposition 1

3.5 The optimal limit with small politician bias

We now turn to the more challenging case in which b is smaller than $(\overline{\sigma} - \sigma_m)/\underline{\sigma}$. We first prove that, under our assumptions on the distribution function H, the optimal limit is never smaller than σ_m.

Lemma 1 *In the static model, the optimal limit is always at least as big as* σ_m.

We now characterize the solution when the biased politician's bias is smaller than $(\overline{\sigma} - \sigma_m)/\underline{\sigma}$ but larger than $(\overline{\sigma} - \sigma_m)/\overline{\sigma}$. In this range of bias levels, with a limit equal to σ_m, the politician will choose his preferred spending level $(1+b)\sigma y$ when σ is sufficiently low but will always choose the spending level $(2\sigma - \sigma_m)y$ when σ exceeds σ_m.

Proposition 2 *In the static model, if the biased politician's bias b lies between $(\overline{\sigma} - \sigma_m)/\overline{\sigma}$ and $(\overline{\sigma} - \sigma_m)/\underline{\sigma}$, the optimal limit solves the equation*

$$1 + H\left(\frac{l}{1+b}\right) = 2H(l). \tag{6}$$

It is straightforward to show that there must exist a solution to equation (6) on the interval $(\sigma_m, \overline{\sigma}]$.[23] While there is no guarantee that this solution is unique, it is difficult to come up with examples satisfying our assumptions in which there are multiple solutions. Figure 2 illustrates a situation in which there exists a unique solution. The Figure depicts the curves $2H(l)$ and $1 + H(l/(1+b))$ on the interval $[\sigma_m, \overline{\sigma}]$. The curve $2H(l)$ must be concave since, under our assumptions, H is concave on $[\sigma_m, \overline{\sigma}]$. The curve $1 + H(l/(1+b))$ is convex on the interval $[\sigma_m, (1+b)\sigma_m]$ and concave thereafter. This follows from the fact that H is convex on $[\underline{\sigma}, \sigma_m]$. As illustrated, the end points and shapes of the two curves guarantee they intersect, with $2H(l)$ intersecting $1 + H(l/(1+b))$ from below.[24]

In the case covered by Proposition 2, the optimal limit becomes more stringent as the biased politician's bias increases. An increase in b shifts down the curve $1 + H(l/(1+b))$. If there is a single intersection point, it must shift to the left, implying a lower optimal limit.[25]

Finally, we tackle the case in which the biased politician's bias is smaller than $(\overline{\sigma} - \sigma_m)/\overline{\sigma}$. In this range of bias levels, with a limit equal to σ_m, the politician will not only choose his preferred spending level $(1+b)\sigma y$ when σ is sufficiently low but will also do so when σ is sufficiently high.

Proposition 3 *In the static model, if the biased politician's bias b is smaller than $(\overline{\sigma} - \sigma_m)/\overline{\sigma}$, the optimal limit is* **either** *bigger than $(1-b)\overline{\sigma}$ and solves equation (6)* **or** *is smaller than $(1-b)\overline{\sigma}$ and solves the equation*

$$H\left(\frac{l}{1-b}\right) + H\left(\frac{l}{1+b}\right) = 2H(l). \tag{7}$$

As noted above, there must exist a solution to equation (6) on the interval $(\sigma_m, \overline{\sigma}]$. We show in the proof of Proposition 3, that if this solution is smaller than $(1-b)\overline{\sigma}$, then there must be

[23] This is shown in the proof of Proposition 2.

[24] If there is more than one solution to equation (6), the end points and shapes of the two curves imply that there must be three. The optimal limit is either the smallest or the largest intersection point, where $2H(l)$ intersects $1 + H(l/(1+b))$ from below.

[25] If there are multiple intersection points, both the smallest and largest shift down. The only way that an increase in b could increase the optimal limit, therefore, is if it caused a shift from the smallest intersection point to the largest. But it is easy to see that an increase in b makes a move from the smallest intersection point to the largest less attractive.

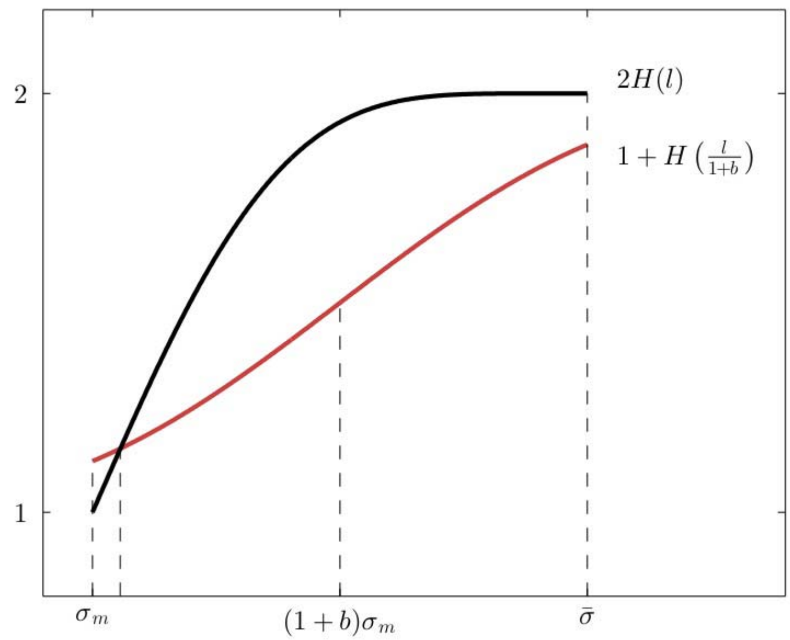

Figure 2: Illustration of Proposition 2

a solution to equation (7) which is also smaller than $(1-b)\overline{\sigma}$. Again, there is no guarantee that there exists a unique solution, but multiple solutions do not arise in examples. Figure 3 graphs the three curves $2H(l)$, $1 + H(l/(1+b))$ and $H(l/(1-b)) + H(l/(1+b))$. The curve $H(l/(1-b)) + H(l/(1+b))$ coincides with $1 + H(l/(1+b))$ at limits higher than $(1-b)\overline{\sigma}$ and lies below it for lower limits. Over this range, it has a steeper slope. The Figure illustrates a situation in which the solution to equation (6) is smaller than $(1-b)\overline{\sigma}$. The optimal limit is therefore found where the curves $2H(l)$ and $H(l/(1-b)) + H(l/(1+b))$ intersect.

It is important to note that in the case in which the optimal limit is found where the curves $2H(l)$ and $H(l/(1-b)) + H(l/(1+b))$ intersect, it is not necessarily the case that increasing the biased politician's bias will make the optimal limit more stringent. This is because an increase in b has ambiguous effects on the curve $H(l/(1-b)) + H(l/(1+b))$. While an increase in b reduces $H(l/(1+b))$ for all l, it increases $H(l/(1-b))$. The net effect is ambiguous.[26] The next sub-

[26] The derivative of the function implicitly defined by the equation $H(\frac{l}{1-b}) + H\left(\frac{l}{1+b}\right) = 2H(l)$ is given by

section provides examples which illustrate that the stringency of the optimal limit can be both decreasing and increasing in bias.

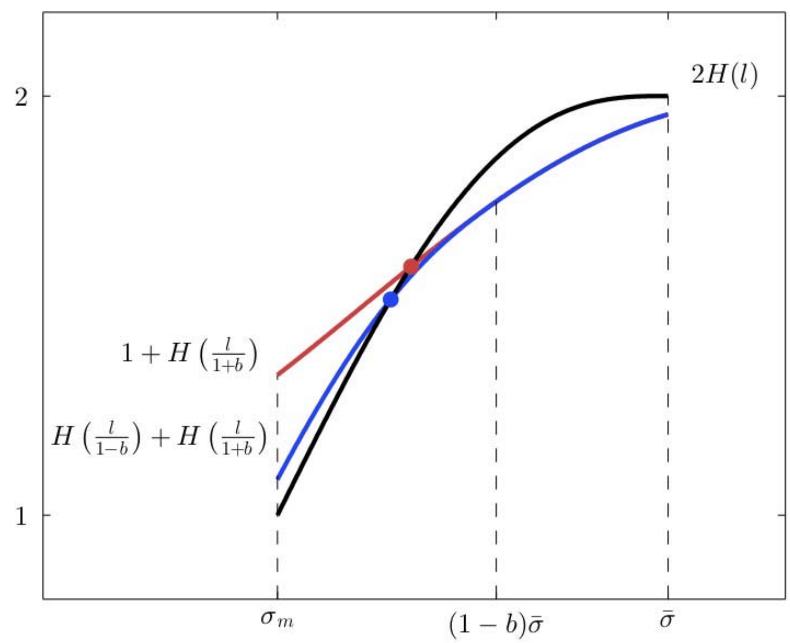

Figure 3: Illustration of Proposition 3

3.6 Examples

We now provide three examples of specific distributions H satisfying our assumptions and use these to explore how the optimal limit depends on the biased politician's bias and the extent of uncertainty in the voter's preferred spending level.

Uniform distribution Suppose that the distribution of the voter's preferred fraction of community income to devote to public spending is uniform; i.e., $H(\sigma)$ equals $(\sigma - \underline{\sigma}) / (\overline{\sigma} - \underline{\sigma})$. Then,

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$$\frac{dl}{db} = \frac{h(\frac{l}{1-b})\frac{l}{(1-b)^2} - h\left(\frac{l}{1+b}\right)\frac{l}{(1+b)^2}}{2h(l) - \frac{h(\frac{l}{1-b})}{1-b} - \frac{h\left(\frac{l}{1+b}\right)}{1+b}}.$$

Assuming that $2H(l)$ intersects $H(l/(1-b)) + H\left(l/(1+b)\right)$ from below, the denominator in this expression will be positive. However, the sign of the numerator is ambiguous.

when the politician's bias b is less than $(\sigma_m - \underline{\sigma})/\underline{\sigma}$, the optimal limit is

$$l(b) = \overline{\sigma}\left(\frac{1+b}{1+2b}\right). \tag{8}$$

To see this, note first that equation (7) has no solution in the uniform case. It therefore follows from Proposition 2 and 3 that the optimal limit must satisfy equation (6). Solving this equation yields (8).

Equation (8) implies that as b approaches 0, the limit converges to $\overline{\sigma}$ and so the politician is completely unconstrained. At first glance, this seems natural, because the politician is becoming a perfect agent for the voter and there is little gain from constraining him. However, the possibility of overrides mean that the limit is irrelevant when the politician is a perfect agent of the voter and therefore it is not clear to what point the limit will converge. Note also from (8) that the limit gets progressively tighter as we increase b, until the point at which b equals $(\sigma_m - \underline{\sigma})/\underline{\sigma}$ and the limit equals the expected preferred fraction σ_m. Further increases in bias have no impact on the limit beyond this point. Figure 4 illustrates the optimal limit as a function of b for the case in which $(\underline{\sigma}, \overline{\sigma})$ equals $(0.1, 0.3)$. The solution is the curve that takes on the value of 0.3 when b is equal to 0 and equals 0.2 for b greater than 0.1.

To understand the impact of changing the distribution of preferred spending levels, it is instructive to consider a parameterization in which $(\underline{\sigma}, \overline{\sigma})$ equals $(\eta - z, \eta + z)$. As we increase z, we hold constant the voter's expected preferred fraction but increase the dispersion. We therefore implement a mean preserving spread. Proposition 1 and (8) tell us that the optimal limit is

$$l(b) = \begin{cases} \frac{(\eta+z)(1+b)}{1+2b} & \text{if } b \in [0, \frac{2z}{2(\eta-z)}] \\ \eta & \text{if } b > \frac{2z}{2(\eta-z)} \end{cases}. \tag{9}$$

These limits are illustrated in Figure 4 for η equal to 0.2 and various values of z. The main point to take away is that with a mean preserving spread, the limit becomes more permissive when the bias of the politician is not too large.

Tent distribution Suppose that the distribution of the voter's preferred fraction is a tent distribution; i.e.,

$$H(\sigma) = \begin{cases} \frac{(\sigma-\underline{\sigma})^2}{2(\sigma_m-\underline{\sigma})^2} & \text{if } \sigma \in [\underline{\sigma}, \sigma_m] \\ \frac{1}{2} + \frac{(\sigma-\sigma_m)[2\overline{\sigma}-(\sigma+\sigma_m)]}{2(\sigma_m-\underline{\sigma})^2} & \text{if } \sigma \in [\sigma_m, \overline{\sigma}] \end{cases}. \tag{10}$$

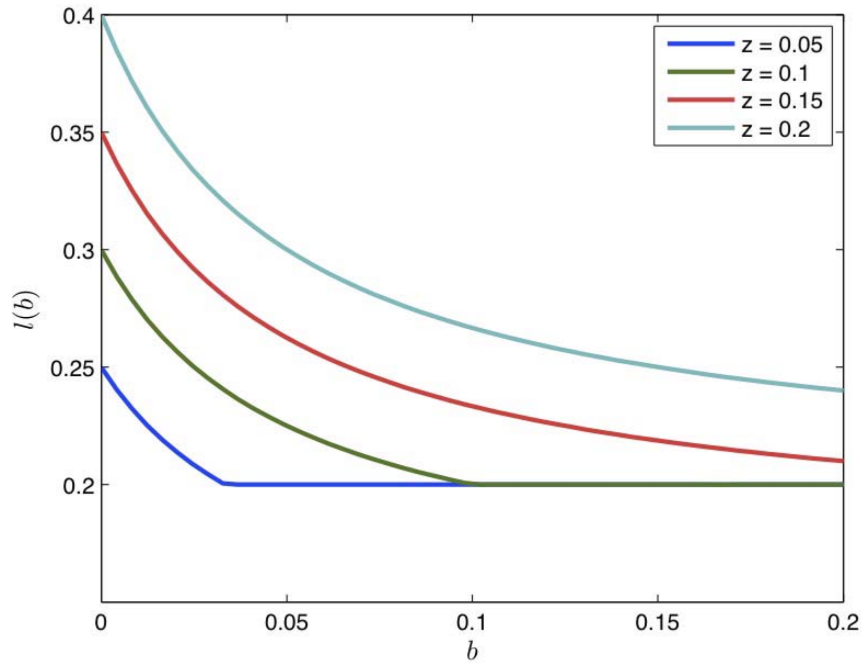

Figure 4: Optimal Limits for the Uniform Distribution

The density associated with this distribution rises linearly from 0 to $1/\left(\sigma_m - \underline{\sigma}\right)$ over the interval $[\underline{\sigma}, \sigma_m]$ and comes back down the other side. Despite its simplicity, this case turns out to be very complicated. Thus, to simplify and permit comparison with the uniform case, we set $(\underline{\sigma}, \overline{\sigma})$ equal to $(0.1, 0.3)$.

In this case, for levels of bias less than 0.03 the optimal limit occurs where the curves $H\left(l/(1-b)\right) + H\left(l/(1+b)\right)$ and $2H(l)$ intersect. For higher levels of bias, the optimal limit occurs where the curves $1 + H\left(l/(1+b)\right)$ and $2H(l)$ intersect. Solving the appropriate quadratic equations reveals that the optimal limit is

$$l(b) = \begin{cases} \dfrac{12+\frac{2}{1+b}-\frac{6}{1-b}-\sqrt{\left(12+\frac{2}{1+b}-\frac{6}{1-b}\right)^2-32\left(2+\frac{1}{(1+b)^2}-\frac{1}{(1-b)^2}\right)}}{2\left(2+\frac{1}{(1+b)^2}-\frac{1}{(1-b)^2}\right)} & \text{if } b \in (0, 0.03) \\ \dfrac{\frac{2}{1+b}+12-\sqrt{8+\frac{48}{1+b}-\frac{64}{(1+b)^2}}}{4+\frac{2}{(1+b)^2}} & \text{if } b \in (0.03, 0.1) \end{cases} \tag{11}$$

This optimal limit is graphed in Figure 5. It is the higher of the two curves that take on the value 0.2 when b is equal to 0. It is instructive to compare this with the optimal limit in the uniform case when $(\underline{\sigma}, \overline{\sigma})$ equals $(0.1, 0.3)$ (i.e., the curve in Figure 4 that takes on the value of 0.3 when b is equal to 0). What is striking is that the optimal limit is much more stringent with the tent distribution. Moreover, the optimal limit does not become more permissive as the politician becomes less biased. Indeed, to the contrary, it becomes more stringent over some part of the range! Analytically, this reflects the fact that the equation determining the optimal limit switches from (6) to (7) as the politician becomes less biased.

What happens when we do a mean preserving spread in the voter's preferred fraction? Figure 5 illustrates the optimal limits for the cases in which $(\underline{\sigma}, \overline{\sigma})$ equals $(0.2 - z, 0.2 + z)$ for various values of z. Note first that the lesson from the uniform case remains: a mean preserving spread results in the optimal limit becoming more permissive when the bias of the politician is not too large. Second, note that as we spread out the distribution, the non-monotonicity exhibited in the case $(\underline{\sigma}, \overline{\sigma})$ equals $(0.1, 0.3)$ disappears. This makes sense intuitively because as we flatten out the tent distribution, it approaches a uniform distribution.

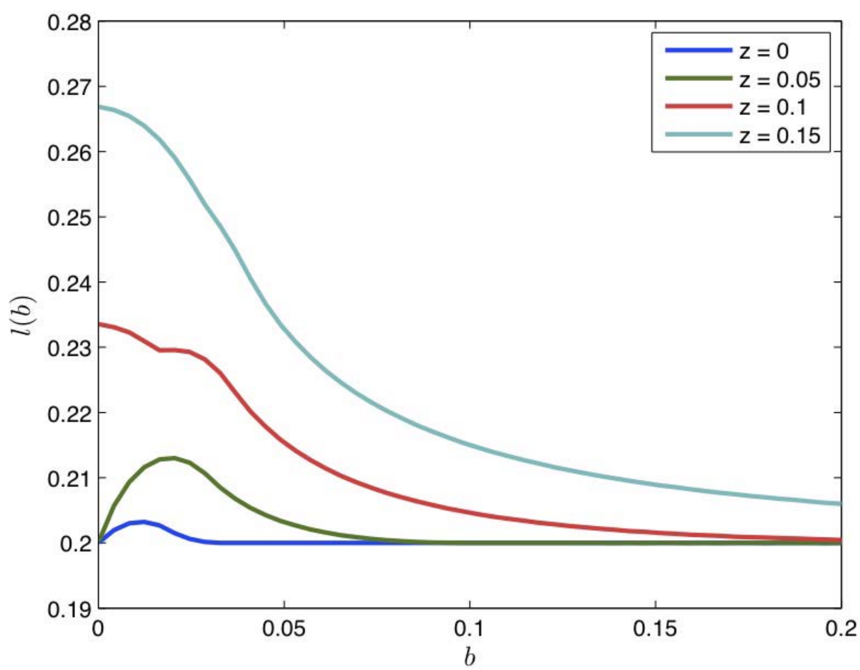

Figure 5: Optimal Limits for the Tent Distribution

Symmetric Beta distribution Suppose that $[\underline{\sigma}, \overline{\sigma}]$ equals $[0, 0.2]$ and that the distribution of the preferred fraction is a symmetric Beta distribution

$$H(\sigma; v) = \frac{\int_0^{\sigma} x^{v-1}(0.2 - x)^{v-1}dx}{\int_0^{0.2} x^{v-1}(0.2 - x)^{v-1}dx}, \tag{12}$$

for v greater than or equal to 1. Recall that when v equals 1 this distribution is just the uniform distribution and when v equals 2 it is the parabolic distribution. As we continue to increase v, probability mass becomes more and more concentrated around the mean 0.1.

Figure 6 graphs the optimal limit as a function of b for various values of v.[27] There are two points to note. First, for given v, the optimal limit is decreasing and approaches the expected preferred spending level σ_m which equals 0.1. It does not quite reach the mean because $\underline{\sigma}$ equals 0 and so Proposition 1 never applies. Thus, we do not get the non-monotonicity displayed in the case of the tent distribution. Second, for given b, as v becomes higher, the optimal limit becomes smaller. Thus, less discretion is provided to the politician as uncertainty is reduced. This is consistent with the findings from the two previous examples, because a move from a symmetric Beta distribution with a higher to a lower v amounts to implementing a mean preserving spread.

Discussion There are two main points to take away from these examples. First, as we increase the uncertainty in the voter's preferred fraction, the optimal limit becomes more permissive at least for bias levels that are not too large. Related comparative static findings have been shown in a number of papers in the literature on the delegation problem. Indeed, Huber and Shipan (2006) refer to the idea that the optimal permissible set of actions for the agent is increased when the principal faces more uncertainty as the *Uncertainty Principle*. It is interesting that this principle continues to hold even though the agent can opt out of the permissible set of actions with the principal's approval. Second, as we reduce the biased politician's bias it is not necessarily the case that the optimal limit becomes more permissive. This contrasts with findings in the literature on the delegation problem which show that the optimal permissible set of actions for the agent is increased when bias is reduced (the so-called *Ally Principle*). The key to understanding this difference is that in our model, because of the override, the limit is irrelevant when the politician is unbiased. Thus, it is not obvious to what point the optimal limit will converge when the biased politician's bias becomes small.

[27] A closed form solution is not available for this case, so the optimal limit is obtained numerically.

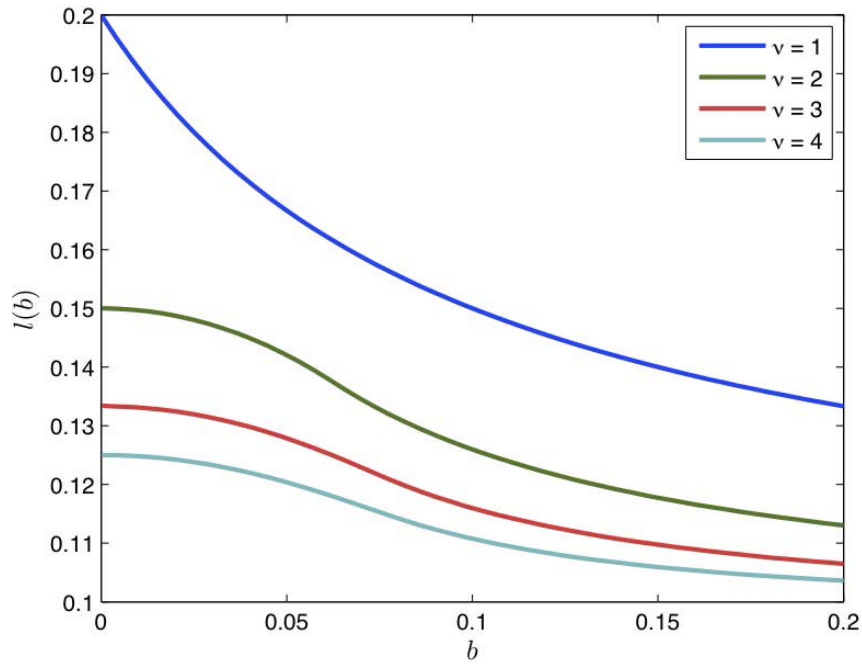

Figure 6: Optimal Limits for the Beta Distribution

3.7 The role of the override provision

From the outset, we have assumed that the limit can be overriden with voter approval. This is because such override provisions are standard in practice. However, it is worth briefly discussing how the voter actually benefits from the override provision.

When the politician is biased, the potential for benefit arises in the situation where the voter's preferred fraction σ exceeds the optimal limit l *and* the maximum spending level the voter will support, $(2\sigma - l)\,y$, exceeds the politician's preferred spending level $(1+b)\sigma y$. If $2\sigma - l$ is smaller than $(1+b)\sigma$, then the politician will propose spending level $(2\sigma - l)\,y$ which, by construction, provides the voter with exactly the same utility level as if the spending level were set equal to the maximum allowed under the limit ly. Notice that in order for it to be possible for $2\sigma - l$ to be larger than $(1+b)\sigma$, it is necessary that $2\overline{\sigma} - l$ is larger than $(1+b)\overline{\sigma}$ which requires that the optimal limit l is less than $(1-b)\overline{\sigma}$. From Propositions 1-3, this requires that b be less than $(\overline{\sigma} - \sigma_m)/\overline{\sigma}$ *and* that the optimal limit satisfies (7). Thus, for a broad range of bias parameters, the voter obtains no benefit from the override provision when the politician is biased. All the surplus it creates goes to the politician. Moreover, in examples such as the uniform distribution analyzed above, the override provision provides no benefit to voters for *all* bias parameters when the politician is biased.

By contrast, when the politician is unbiased, the override provision is *always* beneficial for the voter. With an override, the unbiased politician always chooses the voter's preferred spending level σy. Without an override, the unbiased politician would have no option but to choose spending level ly when the voter's preferred fraction σ exceeds l. The main point to take away, therefore, is that it is the presence of unbiased politicians that, in this model, guarantee that there is a benefit to the voter from having an override provision.[28]

4 Dynamic analysis

We now extend the analysis to a dynamic setting. The dynamic problem is harder to analyze and a complete characterization of optimal limits in this setting is beyond the scope of this paper. Rather our goal is to make a specific point about the desirability of schemes that limit the rate of growth of spending as opposed to those which cap spending below a fixed fraction of community income. To make this point as simply as possible, we consider a two period extension of our model.

4.1 The dynamic model

There are 2 periods indexed by $t \in \{1, 2\}$. The level of spending and community income in period t are denoted s_t and y_t. The voter's preferred spending level in period t is $\sigma_t y_t$. In each period t, the voter has distance preferences $-|s_t - \sigma_t y_t|$. The voter discounts future payoffs at rate β.

The voter's preferred fraction of community income to devote to public spending in period t σ_t is assumed to evolve according to the stochastic process

$$\sigma_t = \sigma_{t-1} + \varepsilon_t, \tag{13}$$

where ε_t is a shock uniformly distributed on the interval $[-\overline{\varepsilon}, \overline{\varepsilon}]$. At the beginning of period 1, the prior period's preferred fraction σ_0 is known, and, to guarantee that σ_t is always between 0 and 1, is assumed to belong to the interval $(2\overline{\varepsilon}, 1 - 2\overline{\varepsilon})$. Specification (13) implies that shocks are

[28] More generally, in a model with a continuous distribution of politician biases, the benefit from the override provision to the voter seems likely to come primarily from those politician types with lower bias.

persistent in the sense that a positive shock in period 1 (i.e., $\varepsilon_1 > 0$) not only increases the voter's preferred fraction in period 1 but also increases it in period 2. This persistence creates a dynamic linkage across periods.

A different politician chooses policy in each period. With probability π the period t politician is biased and has policy preferences $-|s_t - (1+b)\,\sigma_t y_t|$ and with probability $1-\pi$ he is unbiased and shares the voter's preferences. In either case, the period t politician cares only about the period t policy choice and so behaves in a myopic manner. To focus the analysis on the main point we want to make, we make the following assumption:

Assumption 1 *The bias parameter b exceeds $2\overline{\varepsilon}/\left(\sigma_0 - 2\overline{\varepsilon}\right)$.*

To relate this assumption to our analysis of the static model, note that it guarantees that, in each period t, b exceeds $\left(\overline{\sigma}_t - \underline{\sigma}_t\right)/\underline{\sigma}_t$. This is because specification (13) implies that $\overline{\sigma}_t - \underline{\sigma}_t$ is equal to $2\overline{\varepsilon}$ and that $\underline{\sigma}_2$ is equal to $\sigma_0 - 2\overline{\varepsilon}$.

At the beginning of period 1, knowing σ_0, the voter selects limits for the two periods. The limit in period t, l_t, specifies the maximal fraction of the community's income that can be devoted to public spending in period t. Within periods, the timing of events is as in the static model. Each period t begins with the current limit l_t and the previous period's preferred fraction σ_{t-1}. Nature determines the period t preferred fraction σ_t by choosing the preference shock ε_t according to equation (13). Then, the politician proposes a spending level s_t. If this proposed policy does not violate the limit, it is implemented. If it exceeds the limit, an election is held. If the voter votes for the proposal, it is implemented. If he votes against, the politician chooses another policy s_t' which respects the limit and this policy is implemented.

4.2 The dynamic limit design problem

Consider the problem of choosing a pair of limits $\{l_1, l_2\}$ to maximize the voter's welfare. To pose the problem formally, the initial step is to understand what happens for any given limit sequence $\{l_1, l_2\}$.

Consider first what happens in period 2. From our analysis of the static model, if the period 2 politician is biased, the spending level implemented will be $\min\{l_2 y_2, (1+b)\sigma_2 y_2\}$ if the voter's preferred fraction σ_2 is less than the limit l_2 and $\min\{\left(2\sigma_2 - l_2\right) y_2, (1+b)\sigma_2 y_2\}$ otherwise. If the period 2 politician is unbiased, the period 2 policy will just equal $\sigma_2 y_2$. Given (13), the voter's expected period 2 welfare conditional on the voter's period 1 preferred fraction σ_1 will be given by $\pi y_2 V(l_2|\,\sigma_1)$ where:

$$V(l_2|\,\sigma_1) = -\left\{\begin{array}{ll} \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} (\min\{2\,(\sigma_1+\varepsilon) - l_2, (1+b)\,(\sigma_1+\varepsilon)\} - (\sigma_1+\varepsilon))\frac{d\varepsilon}{2\overline{\varepsilon}} & \text{if } l_2 < \sigma_1 - \overline{\varepsilon} \\ \int_{-\overline{\varepsilon}}^{l_2-\sigma_1} (\min\{l_2, (1+b)\,(\sigma_1+\varepsilon)\} - (\sigma_1+\varepsilon))\frac{d\varepsilon}{2\overline{\varepsilon}} & \\ \quad + \int_{l_2-\sigma_1}^{\overline{\varepsilon}} (\min\{2\,(\sigma_1+\varepsilon) - l_2, (1+b)\,(\sigma_1+\varepsilon)\} - (\sigma_1+\varepsilon))\frac{d\varepsilon}{2\overline{\varepsilon}} & \text{if } l_2 \in [\sigma_1 - \overline{\varepsilon}, \sigma_1 + \overline{\varepsilon}] \\ \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} (\min\{l_2, (1+b)\,(\sigma_1+\varepsilon)\} - (\sigma_1+\varepsilon))\frac{d\varepsilon}{2\overline{\varepsilon}} & \text{if } l_2 > \sigma_1 + \overline{\varepsilon} \end{array}\right. . \tag{14}$$

This expression is more complicated than that for the static case (i.e., expression (2)) because there is no guarantee that the second period limit l_2 belongs to the range of possible values for σ_2 (which is $[\sigma_1 - \overline{\varepsilon}, \sigma_1 + \overline{\varepsilon}]$). This reflects the fact that l_2 is chosen before the realization of the voter's period 1 preferred fraction σ_1. A particularly high realization of σ_1 may result in the limit being smaller than $\sigma_1 - \overline{\varepsilon}$. This case is represented by the first part of the expression in (14). Similarly, a particularly low realization may result in the limit being higher than $\sigma_1 + \overline{\varepsilon}$, a case represented by the third component of (14).

Now consider what happens in period 1. Suppose first the period 1 politician is biased. The period 1 policy choice has no implications for what happens in period 2 and so the politician behaves as in the static model. Thus, the policy implemented will be $\min\{l_1 y_1, (1+b)\sigma_1 y_1\}$ if the voter's preferred fraction σ_1 is less than the limit l_1 and $\min\{(2\sigma_1 - l_1)\,y_1, (1+b)\sigma_1 y_1\}$ otherwise. If the period 1 politician is unbiased, the period 1 policy will just equal $\sigma_1 y_1$. The voter's expected period 1 welfare conditional on σ_0 will therefore be given by $\pi y_1 V(l_1|\,\sigma_0)$, where the function $V(l|\,\sigma)$ is as defined in (14). Note however that because σ_0 is known at the time the limits are set at the beginning of period 1 we may assume that l_1 belongs to the range of possible values for the voter's period 1 preferred fraction σ_1 (which is $[\sigma_0 - \overline{\varepsilon}, \sigma_0 + \overline{\varepsilon}]$).

The dynamic limit design problem is to choose a sequence of limits $\{l_1, l_2\}$ to solve

$$\max_{\{l_1,l_2\}} \pi \left[y_1 V(l_1|\,\sigma_0) + \beta y_2 \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,(\sigma_0+\varepsilon_1))\frac{d\varepsilon_1}{2\overline{\varepsilon}} \right]. \tag{15}$$

The following proposition tells us that for bias levels satisfying Assumption 1, the optimal policy is to set both periods' limits equal to the voter's expected preferred fraction conditional on the information available at the beginning of period 1 (i.e., σ_0).

Proposition 4 *In the dynamic model, under Assumption 1, the optimal sequence of limits is*

$\{\sigma_0, \sigma_0\}$.

Proposition 4 is the natural dynamic generalization of Proposition 1. As noted in the introduction, schemes that cap spending to be lower than some constant fraction of community income are used in practice. This proposition tells us what this fraction should be - it is the optimal fraction of income that should be devoted to public spending in the period prior to the system being introduced.

4.3 Spending-contingent limits

The above analysis assumes that the period 2 limit must be set in stone at the beginning of time 1. This creates a welfare loss in period 2 relative to the static model because the period 2 limit does not reflect the voter's period 1 preference shock. Implicit in the analysis is the assumption that the period 2 limit cannot be made conditional on the voter's period 1 preferred fraction σ_1. The motivation for this assumption is that this preferred fraction reflects the voter's preference shock ε_1 and this depends on too many factors to be coded into a limit formula. Nonetheless, it is perfectly possible to make the period 2 limit depend upon the *actual* period 1 spending level s_1. Our next proposition shows that making the period 2 limit contingent on the fraction of community income devoted to public spending in period 1 results in a welfare improvement for the voter.

Proposition 5 *In the dynamic model, under Assumption 1, the sequence of spending-contingent limits* $\{\sigma_0, s_1/y_1\}$ *yields the voter a higher level of expected welfare than the optimal non spending-contingent sequence* $\{\sigma_0, \sigma_0\}$.

The policy described in Proposition 5 requires that spending in the initial period not exceed a fraction σ_0 of the community's income and that, thereafter, spending not exceed the fraction of community income that it was in the prior period. The latter requirement amounts to the restriction that the growth rate of spending, defined as $(s_t - s_{t-1})/s_{t-1}$, should not exceed the growth rate of community income which is $g_t = (y_t - y_{t-1})/y_{t-1}$.[29] As noted in the introduction, spending limits that limit the growth rate of spending to a rate tied to the growth of community income, population, etc, are common in practice. The dynamic analysis therefore tells us that this

[29] To see this, note that the policy requires that in period 2 s_2/y_2 is less than s_1/y_1. By definition, $y_2 = (1+g_2)y_1$ and hence the policy requires that $(s_2 - s_1)/s_1$ is less than g_2.

class of spending limits is superior to the class that cap spending at some fixed fraction of community income. This said, it is important to stress that even with a policy that limits the growth of spending, a cap must be applied in the initial period to move spending to a lower trajectory. Assuming, as seems reasonable, that spending in the period before limits are introduced exceeds the voter's preferred level (i.e., s_0/y_0 exceeds σ_0), a growth based restriction on spending in the initial period will be insufficiently stringent.

To understand Proposition 5 intuitively, recall that the problem with the cap policy of Proposition 4 is that the period 2 limit is not sensitive to the voter's preference shock in period 1. It may then diverge markedly from the optimal period 2 limit which, by Proposition 1, is σ_1. By contrast, under the policy that limits growth, the period 2 limit will be exactly σ_1 if the period 1 politician is unbiased. This is because such a politician will choose the voter's preferred spending level $\sigma_1 y_1$. The voter is therefore strictly better off under the growth limiting policy if the period 1 politician is unbiased. Less obviously, the voter must be at least as well off with the growth policy if the period 1 politician is biased. Such a politician will either choose a spending level equal to the limit $\sigma_0 y_1$ or he will choose some larger spending level. In the former case, the period 2 limit will be σ_0 as in the cap case and there is no difference between the two regimes. In the latter case, it is unclear what the period 2 limit will be. However, given that the voter has chosen to approve the period 1 spending level, his two period expected payoff must be at least as large as that resulting if the period 1 politician just selected $\sigma_0 y_1$ and the period 2 limit were σ_0. Moreover, under the cap policy, this is precisely the payoff he would obtain in equilibrium.

5 Conclusion

This paper has presented a simple political economy model in which to study the optimal design of fiscal limits. Reflecting common practice, the focus has been on limits that cap the maximal fraction of community income that can be devoted to public spending and which can be overriden with voters' approval. The analysis of the static version of this model sheds light on how the optimal limit depends on the level of bias towards spending in the political system and the nature of uncertainty concerning voters' spending preferences. When the level of bias is high, the optimal limit just equals the fraction of community income the representative voter expects to want to devote to public spending. For smaller levels of bias, more permissive limits are optimal. However, it is not necessarily the case that limits become increasingly more permissive as the level of bias falls. This will depend on the distribution of preferred spending levels. Our examples suggest that, for low bias levels, more uncertainty in the voter's preferred spending level results in a more permissive optimal limit.

The analysis of the dynamic version of the model sheds light on how limits should evolve over time. The dynamic model assumes that the fraction of community income the representative voter would like to see devoted to public spending evolves according to a stochastic process with persistent shocks. In this setting, systems that limit the growth of spending dominate those that just cap spending below some constant fraction of community income. Given that both systems are employed in practice, this is an interesting finding.

While this paper has hopefully made a significant contribution in highlighting the optimal fiscal limit problem and providing some understanding of the nature of the solution, there is scope for considerably more work on this practically important and theoretically interesting problem. Extensions of the static model readily suggest themselves. It would be interesting to go beyond the simple preferences assumed here by, for example, introducing convexity into the voter's loss function. We could then understand how greater convexity influences the permissiveness of the limit. Relaxing the assumptions on the distribution of preferred spending levels may also prove instructive. Richer uncertainty in the degree of politician bias could also be introduced. Finally, recognizing that it is costly to hold an override election might yield interesting results.

Everything that could be done to the static model could of course be done to the dynamic model. In addition, extensions that generalize the dynamic structure such as adding more periods or changing how preferences evolve over time readily suggest themselves. However, prior to such extensions, it would be useful to have a full characterization of optimal limits in the simple dynamic model of Section 4.1. This would include extending the analysis to cover smaller bias levels (i.e., relaxing Assumption 1) and understanding precisely how limits should be tied to spending outcomes in past periods. That is, while Proposition 5 tells us that spending-contingent limits are optimal, is the best form of spending-contingent limit that described in Proposition 5 or are there better options?

Beyond these conceptually straightforward extensions, considering optimal limits in a legislative setting would be interesting. In such a setting, spending is determined by the collective decisions of legislators rather than the decision of a single politician. Consistent with practice, it would be natural to consider override provisions which allow the limit to be overriden by a super-majority of legislators rather than by direct appeal to the voters. Continually undertaking spending referenda is likely to prove administratively costly and it may be that the same function can be achieved by appropriate choice of super-majority override.

Of course, even if all these extensions were undertaken, the analysis would only provide insight into what limits should depend on *in principle.* Moving in the direction of being able to say what limits should be in concrete situations would require effort in two directions. First, developing models like the ones presented here into credible models for thinking about limits empirically. This requires developing models whose testable implications are consistent with the data. Such implications would concern the dynamic path of spending and the use of overrides. Second, with such models in hand, trying to measure the key determinants of the optimal limit. For example, how could we measure the extent of politicians' bias and the nature of uncertainty in voters' preferred spending levels?

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6 Appendix

6.1 Proof of Proposition 1

We need to show that $V(\sigma_m)$ exceeds $V(l)$ for any limit l in the range $[\underline{\sigma}, \sigma_m)$ or $(\sigma_m, \overline{\sigma}]$. Since b exceeds $(\overline{\sigma} - \sigma_m)/\underline{\sigma}$, $l/(1+b)$ is less than $\underline{\sigma}$ for any limit l in the range $[\underline{\sigma}, \sigma_m]$ and, if b is less than 1, $l/(1-b)$ exceeds $\overline{\sigma}$ for any limit l in the range $[\sigma_m, \overline{\sigma}]$. Thus, from (3), we have that

$$V(\sigma_m) = \int_{\underline{\sigma}}^{\sigma_m} \left[(1+b)\sigma - \sigma_m\right] h(\sigma) d\sigma + \int_{\sigma_m}^{\overline{\sigma}} \left[\sigma_m - (1-b)\sigma\right] h(\sigma) d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma) d\sigma. \tag{16}$$

Recall from the analysis in the text, that we know already that for any l in the range $[\underline{\sigma}, \sigma_m)$ or $(\sigma_m, \overline{\sigma}]$ we have that

$$V(\sigma_m) > \int_{\underline{\sigma}}^{l} \left[(1+b)\sigma - l\right] h(\sigma) d\sigma + \int_{l}^{\overline{\sigma}} \left[l - (1-b)\sigma\right] h(\sigma) d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma) d\sigma. \tag{17}$$

Consider a limit l in the range $[\underline{\sigma}, \sigma_m)$. If b exceeds 1 or if $l/(1-b)$ exceeds $\overline{\sigma}$, then from (3), $V(l)$ is equal to the right hand side of (17) and thus the desired inequality holds. If $l/(1-b)$ is less than $\overline{\sigma}$, then from (3), we have that

$$V(l) = \int_{\underline{\sigma}}^{l} \left[(1+b)\sigma - l\right] h(\sigma) d\sigma + \int_{l}^{\frac{l}{1-b}} \left[l - (1-b)\sigma\right] h(\sigma) d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma) d\sigma. \tag{18}$$

From (16) and (18), we have that

$$\begin{aligned} V(\sigma_m) - V(l) \quad = \quad & \int_{\underline{\sigma}}^{\sigma_m} \left[(1+b)\sigma - \sigma_m\right] h(\sigma) d\sigma + \int_{\sigma_m}^{\overline{\sigma}} \left[\sigma_m - (1-b)\sigma\right] h(\sigma) d\sigma \\ & - \int_{\underline{\sigma}}^{l} \left[(1+b)\sigma - l\right] h(\sigma) d\sigma - \int_{l}^{\frac{l}{1-b}} \left[l - (1-b)\sigma\right] h(\sigma) d\sigma. \end{aligned}$$

We can write this difference as

$$\begin{aligned} V(\sigma_m) - V(l) \quad = \quad & \int_{\underline{\sigma}}^{l} \left[(1+b)\sigma - \sigma_m\right] h(\sigma) d\sigma + \int_{l}^{\sigma_m} \left[(1+b)\sigma - \sigma_m\right] h(\sigma) d\sigma + \\ & \int_{\sigma_m}^{\frac{l}{1-b}} \left[\sigma_m - (1-b)\sigma\right] h(\sigma) d\sigma + \int_{\frac{l}{1-b}}^{\overline{\sigma}} \left[\sigma_m - (1-b)\sigma\right] h(\sigma) d\sigma \\ & - \int_{\underline{\sigma}}^{l} \left[(1+b)\sigma - l\right] h(\sigma) d\sigma - \int_{l}^{\frac{l}{1-b}} \left[l - (1-b)\sigma\right] h(\sigma) d\sigma, \end{aligned}$$

which simplifies to

$$\begin{aligned} V(\sigma_m) - V(l) \quad = \quad & \int_{\underline{\sigma}}^{l} \left[l - \sigma_m\right] h(\sigma) d\sigma + \int_{l}^{\sigma_m} \left[2\sigma - (\sigma_m + l)\right] h(\sigma) d\sigma + \\ & \int_{\sigma_m}^{\frac{l}{1-b}} \left[\sigma_m - l\right] h(\sigma) d\sigma + \int_{\frac{l}{1-b}}^{\overline{\sigma}} \left[\sigma_m - (1-b)\sigma\right] h(\sigma) d\sigma. \end{aligned}$$

This in turn can be rewritten as

$$\begin{aligned}V(\sigma_m)-V(l) &= \int_{\underline{\sigma}}^{\sigma_m}[l-\sigma_m]\,h(\sigma)d\sigma+\int_{l}^{\sigma_m}[2\sigma-(\sigma_m+l)-l+\sigma_m]\,h(\sigma)d\sigma+\\ &\quad \int_{\sigma_m}^{\overline{\sigma}}[\sigma_m-l]\,h(\sigma)d\sigma+\int_{\frac{l}{1-b}}^{\overline{\sigma}}[\sigma_m-(1-b)\sigma-\sigma_m+l]\,h(\sigma)d\sigma,\end{aligned}$$

which simplifies to

$$V(\sigma_m)-V(l)=2\int_{l}^{\sigma_m}[\sigma-l]\,h(\sigma)d\sigma-\int_{\frac{l}{1-b}}^{\overline{\sigma}}[(1-b)\sigma-l]\,h(\sigma)d\sigma.$$

Thus, we need to show that

$$2\int_{l}^{\sigma_m}[\sigma-l]\,h(\sigma)d\sigma>\int_{\frac{l}{1-b}}^{\overline{\sigma}}[(1-b)\sigma-l]\,h(\sigma)d\sigma.$$

Because $h(\sigma)$ is non-decreasing on $[l,\sigma_m]$, we know that

$$2\int_{l}^{\sigma_m}[\sigma-l]\,h(\sigma)d\sigma\geq 2\left[\frac{\sigma_m+l}{2}-l\right]\int_{l}^{\sigma_m}h(\sigma)d\sigma=(\sigma_m-l)\int_{l}^{\sigma_m}h(\sigma)d\sigma.$$

Similarly, because $h(\sigma)$ is non-increasing on $[\frac{l}{1-b},\overline{\sigma}]$, we know that

$$\int_{\frac{l}{1-b}}^{\overline{\sigma}}[(1-b)\sigma-l]\,h(\sigma)d\sigma\leq\left[(1-b)\left(\frac{\overline{\sigma}+\frac{l}{1-b}}{2}\right)-l\right]\int_{\frac{l}{1-b}}^{\overline{\sigma}}h(\sigma)d\sigma=\left(\frac{(1-b)\overline{\sigma}-l}{2}\right)\int_{\frac{l}{1-b}}^{\overline{\sigma}}h(\sigma)d\sigma.$$

Since $\sigma_m\geq(1-b)\overline{\sigma}$, it therefore suffices to show that

$$\int_{l}^{\sigma_m}h(\sigma)d\sigma\geq\int_{\frac{l}{1-b}}^{\overline{\sigma}}h(\sigma)d\sigma.$$

Given the assumed properties of h, a sufficient condition for this is that

$$\sigma_m-l\geq\overline{\sigma}-\frac{l}{1-b}\Leftrightarrow\frac{bl}{1-b}>\overline{\sigma}-\sigma_m.$$

But we know that

$$\frac{bl}{1-b}\geq\frac{b\underline{\sigma}}{1-b}\geq\frac{(\overline{\sigma}-\sigma_m)}{1-b}>\overline{\sigma}-\sigma_m.$$

Now consider a limit l in the range $(\sigma_m,\overline{\sigma}]$. If $l/(1+b)$ is less than $\underline{\sigma}$, then from (3), $V(l)$ is equal to the right hand side of (17) and thus the desired inequality holds. If $l/(1+b)$ exceeds $\underline{\sigma}$, then from (3), we have that

$$V(l)=\int_{\frac{l}{1+b}}^{l}[(1+b)\sigma-l]\,h(\sigma)d\sigma+\int_{l}^{\overline{\sigma}}[l-(1-b)\sigma]\,h(\sigma)d\sigma-\int_{\underline{\sigma}}^{\overline{\sigma}}b\sigma h(\sigma)d\sigma. \qquad (19)$$

Note that $l/(1+b) < \sigma_m$ and thus using (16) and (19) we can write

$$\begin{aligned} V(\sigma_m) - V(l) &= \int_{\underline{\sigma}}^{\frac{l}{1+b}} \left[(1+b)\sigma - \sigma_m\right] h(\sigma) d\sigma + \int_{\frac{l}{1+b}}^{\sigma_m} \left[(1+b)\sigma - \sigma_m\right] h(\sigma) d\sigma \\ &+ \int_{\sigma_m}^{l} \left[\sigma_m - (1-b)\sigma\right] h(\sigma) d\sigma + \int_{l}^{\overline{\sigma}} \left[\sigma_m - (1-b)\sigma\right] h(\sigma) d\sigma \\ &- \int_{\frac{l}{1+b}}^{\sigma_m} \left[(1+b)\sigma - l\right] h(\sigma) d\sigma - \int_{\sigma_m}^{l} \left[(1+b)\sigma - l\right] h(\sigma) d\sigma - \int_{l}^{\overline{\sigma}} \left[l - (1-b)\sigma\right] h(\sigma) d\sigma. \end{aligned}$$

This equals

$$\begin{aligned} V(\sigma_m) - V(l) &= \int_{\underline{\sigma}}^{\frac{l}{1+b}} \left[(1+b)\sigma - \sigma_m\right] h(\sigma) d\sigma + \int_{\frac{l}{1+b}}^{\sigma_m} \left[l - \sigma_m\right] h(\sigma) d\sigma + \int_{\sigma_m}^{l} \left[\sigma_m + l - 2\sigma\right] h(\sigma) d\sigma \\ &+ \int_{l}^{\overline{\sigma}} \left[\sigma_m - l\right] h(\sigma) d\sigma, \end{aligned}$$

which simplifies to

$$V(\sigma_m) - V(l) = 2\int_{\sigma_m}^{l} \left[l - \sigma\right] h(\sigma) d\sigma - \int_{\underline{\sigma}}^{\frac{l}{1+b}} \left[l - (1+b)\sigma\right] h(\sigma) d\sigma.$$

This is the difference illustrated in Panel D of Figure 1. Thus, we need to show that

$$2\int_{\sigma_m}^{l} \left[l - \sigma\right] h(\sigma) d\sigma > \int_{\underline{\sigma}}^{\frac{l}{1+b}} \left[l - (1+b)\sigma\right] h(\sigma) d\sigma.$$

Because $h(\sigma)$ is non-increasing on $[\sigma_m, l]$, we know that

$$2\int_{\sigma_m}^{l} \left[l - \sigma\right] h(\sigma) d\sigma \geq 2\left[\frac{l+\sigma_m}{2} - \sigma_m\right] \int_{\sigma_m}^{l} h(\sigma) d\sigma = \left[l - \sigma_m\right] \int_{\sigma_m}^{l} h(\sigma) d\sigma.$$

Similarly, because $h(\sigma)$ is non-decreasing on $[\underline{\sigma}, \frac{l}{1+b}]$, we know that

$$\int_{\underline{\sigma}}^{\frac{l}{1+b}} \left[l - (1+b)\sigma\right] h(\sigma) d\sigma \leq \left[l - (1+b)\left(\frac{\frac{l}{1+b} + \underline{\sigma}}{2}\right)\right] \int_{\underline{\sigma}}^{\frac{l}{1+b}} h(\sigma) d\sigma = \left(\frac{l - (1+b)\underline{\sigma}}{2}\right) \int_{\underline{\sigma}}^{\frac{l}{1+b}} h(\sigma) d\sigma.$$

We know that $\sigma_m \leq (1+b)\underline{\sigma}$, so it suffices to show that

$$\int_{\sigma_m}^{l} h(\sigma) d\sigma \geq \int_{\underline{\sigma}}^{\frac{l}{1+b}} h(\sigma) d\sigma.$$

Given the assumed properties of h, a sufficient condition for this is that

$$l - \sigma_m \geq \frac{l}{1+b} - \underline{\sigma} \Leftrightarrow \frac{bl}{1+b} > \sigma_m - \underline{\sigma}.$$

But we know that

$$\frac{bl}{1+b} \geq \frac{b\underline{\sigma}}{1+b} \geq \frac{(\overline{\sigma} - \sigma_m)}{1+b} > \sigma_m - \underline{\sigma}.$$

■

6.2 Proof of Lemma 1

Proposition 1 implies that the result is true for b larger than $(\overline{\sigma} - \sigma_m)/\underline{\sigma}$. Thus, we just need to show that the result is true for b smaller than $(\overline{\sigma} - \sigma_m)/\underline{\sigma}$. Consider some limit $l < \sigma_m$. We will show that marginally increasing l will increase the voter's payoff.

Suppose first that $l \geq (1-b)\overline{\sigma}$. If $l \geq (1+b)\underline{\sigma}$, then, from (3), we have that

$$V(l) = \int_{\frac{l}{1+b}}^{l} [(1+b)\sigma - l]\, h(\sigma)d\sigma + \int_{l}^{\overline{\sigma}} [l - (1-b)\sigma]\, h(\sigma)d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma)d\sigma.$$

Note that

$$V'(l) = -\int_{\frac{l}{1+b}}^{l} h(\sigma)d\sigma + \int_{l}^{\overline{\sigma}} h(\sigma)d\sigma = 1 + H(\frac{l}{1+b}) - 2H(l) > 0,$$

which implies that raising the limit slightly will increase the voter's payoff. If $l < (1+b)\underline{\sigma}$, then, from (3), we have that

$$V(l) = \int_{\underline{\sigma}}^{l} [(1+b)\sigma - l]\, h(\sigma)d\sigma + \int_{l}^{\overline{\sigma}} [l - (1-b)\sigma]\, h(\sigma)d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma)d\sigma.$$

Note that

$$V'(l) = -\int_{\underline{\sigma}}^{l} h(\sigma)d\sigma + \int_{l}^{\overline{\sigma}} h(\sigma)d\sigma = 1 - 2H(l) > 0,$$

which again implies that raising l marginally benefits the voter.

Now suppose that $l < (1-b)\overline{\sigma}$. If $l \geq (1+b)\underline{\sigma}$, then, from (3), we have that

$$V(l) = \int_{\frac{l}{1+b}}^{l} [(1+b)\sigma - l]\, h(\sigma)d\sigma + \int_{l}^{\frac{l}{1-b}} [l - (1-b)\sigma]\, h(\sigma)d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma)d\sigma.$$

Note that

$$V'(l) = -\int_{\frac{l}{1+b}}^{l} h(\sigma)d\sigma + \int_{l}^{\frac{l}{1-b}} h(\sigma)d\sigma.$$

Given that $\frac{l}{1+b} < l < \sigma_m$ and that

$$l - \frac{l}{1+b} = \frac{bl}{1+b} < \frac{bl}{1-b} = \frac{l}{1-b} - l,$$

the assumption that h is symmetric and non-decreasing on $[\underline{\sigma}, \sigma_m]$ implies that

$$\int_{l}^{\frac{l}{1-b}} h(\sigma)d\sigma > \int_{\frac{l}{1+b}}^{l} h(\sigma)d\sigma.$$

To see this, note that for any $\sigma \in [\frac{l}{1+b}, l]$ we can associate a unique $\sigma' \in [l, \frac{l}{1-b}]$ (e.g., $\sigma' = 2l - x$) which has a higher density. Thus, it must be the case that $V'(l) > 0$ which implies that raising the limit slightly will increase the voter's payoff. If $l < (1+b)\underline{\sigma}$, then, from (3), we have that

$$V(l) = \int_{\underline{\sigma}}^{l} [(1+b)\sigma - l]\, h(\sigma)d\sigma + \int_{l}^{\frac{l}{1-b}} [l - (1-b)\sigma]\, h(\sigma)d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma)d\sigma.$$

Note that

$$V'(l) = -\int_{\underline{\sigma}}^{l} h(\sigma)d\sigma + \int_{l}^{\frac{l}{1-b}} h(\sigma)d\sigma.$$

Given that $\underline{\sigma} < l < \sigma_m$ and that

$$l - \underline{\sigma} < l - \frac{l}{1+b} = \frac{bl}{1+b} < \frac{bl}{1-b} = \frac{l}{1-b} - l,$$

the assumption that h is symmetric and non-decreasing on $[\underline{\sigma}, \sigma_m]$ implies that

$$\int_{l}^{\frac{l}{1-b}} h(\sigma)d\sigma > \int_{\underline{\sigma}}^{l} h(\sigma)d\sigma.$$

Again, to see this note that for any $\sigma \in [\underline{\sigma}, l]$ we can find a unique $\sigma' \in [l, \frac{l}{1-b}]$ (e.g., $\sigma' = 2l - x$) which has a higher density. Thus, $V'(l) > 0$ which again implies that raising l marginally benefits the voter. ■

6.3 Proof of Proposition 2

For limits $l \in [\sigma_m, \overline{\sigma}]$, we have that $l/(1+b)$ is greater than or equal to $\sigma_m/(1+b)$ which, since b is less than $(\sigma_m - \underline{\sigma})/\underline{\sigma}$, exceeds $\underline{\sigma}$. In addition, if $b < 1$, we have that $l/(1-b)$ is greater than or equal to $\sigma_m/(1-b)$ which, since b exceeds $(\overline{\sigma} - \sigma_m)/\overline{\sigma}$, exceeds $\overline{\sigma}$. Thus, for limits $l \in [\sigma_m, \overline{\sigma}]$, (3) implies that

$$V(l) = \int_{\frac{l}{1+b}}^{l} [(1+b)\sigma - l]\, h(\sigma)d\sigma + \int_{l}^{\overline{\sigma}} [l - (1-b)\sigma]\, h(\sigma)d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma)d\sigma.$$

This means that

$$V'(l) = -\int_{\frac{l}{1+b}}^{l} h(\sigma)d\sigma + \int_{l}^{\overline{\sigma}} h(\sigma)d\sigma = 1 + H\left(\frac{l}{1+b}\right) - 2H(l).$$

It follows that at the optimal limit

$$1 + H\left(\frac{l}{1+b}\right) = 2H(l),$$

which is (6). To see that this equation has a solution, note that

$$1 + H(\frac{\sigma_m}{1+b}) > 1 = 2H(\sigma_m),$$

and that

$$1 + H\left(\frac{\overline{\sigma}}{1+b}\right) < 2H(\overline{\sigma}) = 2.$$

Thus, by the *Intermediate Value Theorem*, there exists a solution to equation (6). ■

6.4 Proof of Proposition 3

For limits $l \in [\sigma_m, \overline{\sigma}]$, we have that $l/(1+b)$ is greater than or equal to $\sigma_m/(1+b)$ which, since b is less than $(\overline{\sigma} - \sigma_m)/\overline{\sigma}$, exceeds $\underline{\sigma}$. Moreover, since $\sigma_m/(1-b)$ is less than $\overline{\sigma}$ which is less than $\overline{\sigma}/(1-b)$, we have that

$$\frac{l}{1-b} \gtrless \overline{\sigma} \text{ as } l \gtrless (1-b)\,\overline{\sigma}.$$

It follows from (3) that the voter's welfare with limit $l \in [\sigma_m, \overline{\sigma}]$ is

$$V(l) = \begin{cases} \int_{\frac{l}{1+b}}^{l} [(1+b)\sigma - l]\, h(\sigma)d\sigma + \int_{l}^{\frac{l}{1-b}} [l - (1-b)\sigma]\, h(\sigma)d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma)d\sigma & \text{if } l < (1-b)\,\overline{\sigma} \\ \int_{\frac{l}{1+b}}^{l} [(1+b)\sigma - l]\, h(\sigma)d\sigma + \int_{l}^{\overline{\sigma}} [l - (1-b)\sigma]\, h(\sigma)d\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} b\sigma h(\sigma)d\sigma & \text{if } l \geq (1-b)\,\overline{\sigma} \end{cases}.$$

Thus, the impact on welfare of a small increase in the limit is

$$V'(l) = \begin{cases} H(\frac{l}{1-b}) + H(\frac{l}{1+b}) - 2H(l) & \text{if } l < (1-b)\overline{\sigma} \\ 1 + H(\frac{l}{1+b}) - 2H(l) & \text{if } l \geq (1-b)\overline{\sigma} \end{cases}.$$

It follows that the optimal limit is either such that $l \in [\sigma_m, (1-b)\overline{\sigma})$ and solves

$$H(\frac{l}{1-b}) + H(\frac{l}{1+b}) = 2H(l),$$

or is such that $l \in [(1-b)\overline{\sigma}, \overline{\sigma}]$ and solves

$$1 + H(\frac{l}{1+b}) = 2H(l).$$

It is straightforward to show that at least one of these equations must have a solution in the relevant range. The assumption that b is less than $(\overline{\sigma} - \sigma_m)/\overline{\sigma}$ implies that b is less than $(\sigma_m - \underline{\sigma})/\underline{\sigma}$ which means that $\sigma_m/(1+b)$ exceeds $\underline{\sigma}$. Thus,

$$1 + H(\frac{\sigma_m}{1+b}) > 1 = 2H(\sigma_m).$$

Since

$$1 + H(\frac{\overline{\sigma}}{1+b}) < 2 = 2H(\overline{\sigma}),$$

there exists $l \in (\sigma_m, \overline{\sigma})$ such that

$$1 + H(\frac{l}{1+b}) = 2H(l).$$

Suppose that for all such l it is the case that $l < (1-b)\overline{\sigma}$, then it must be the case that

$$1 + H(\frac{(1-b)\overline{\sigma}}{1+b}) < 2H((1-b)\overline{\sigma}) \Leftrightarrow H(\frac{(1-b)\overline{\sigma}}{1+b}) + H(\frac{(1-b)\overline{\sigma}}{1-b}) < 2H((1-b)\overline{\sigma}).$$

If

$$H(\frac{\sigma_m}{1-b}) + H(\frac{\sigma_m}{1+b}) > 1 = 2H(\sigma_m), \tag{20}$$

this implies that there exists $l \in [\sigma_m, (1-b)\overline{\sigma}]$ such that

$$H(\frac{l}{1-b}) + H(\frac{l}{1+b}) = 2H(l).$$

It suffices, therefore, to prove that (20) holds. Note that symmetry implies that

$$H(\frac{\sigma_m}{1+b}) = 1 - H(\sigma_m + \frac{b\sigma_m}{1+b}).$$

Moreover, we have that

$$\frac{\sigma_m}{1-b} = \sigma_m + \frac{b\sigma_m}{1-b} > \sigma_m + \frac{b\sigma_m}{1+b}.$$

This means that

$$H(\frac{\sigma_m}{1+b}) + H(\frac{\sigma_m}{1-b}) > H(\frac{\sigma_m}{1+b}) + H(\sigma_m + \frac{b\sigma_m}{1+b}) = 1.$$

■

6.5 Proof of Proposition 4

We first show that $l_1 = \sigma_0$. The period 1 limit l_1 has no implications for welfare in period 2 and hence the optimal limit just solves the static problem considered in Section 2 under the assumptions that $\overline{\sigma} = \sigma_0 + \overline{\varepsilon}$, $\underline{\sigma} = \sigma_0 - \overline{\varepsilon}$, and $H(\sigma) = (\sigma - \underline{\sigma})/2\overline{\varepsilon}$. We have that $\sigma_m = \sigma_0$. Thus the result will follow from Proposition 1 if $b > (\overline{\sigma} - \sigma_m)/\underline{\sigma} = \overline{\varepsilon}/(\sigma_0 - \overline{\varepsilon})$. We have that

$$\frac{\overline{\varepsilon}}{\sigma_0 - \overline{\varepsilon}} < \frac{2\overline{\varepsilon}}{\sigma_0 - 2\overline{\varepsilon}} < b,$$

where the last line follows by Assumption 1.

We now show that $l_2 = \sigma_0$. The difficulty in doing this is dealing with the period 2 welfare expressions in (14) and (15) which are unwieldy. The first point to note is that Assumption 1 implies that $\sigma_0 < (b+1)(\sigma_0 - 2\overline{\varepsilon})$ and $\sigma_0 > (1-b)(\sigma_0 + 2\overline{\varepsilon})$. This means that for all $\sigma_1 \in [\sigma_0 - \overline{\varepsilon}, \sigma_0 + \overline{\varepsilon}]$ and $\varepsilon_2 \in [-\overline{\varepsilon}, \overline{\varepsilon}]$, $\sigma_0 < (b+1)[\sigma_1 + \varepsilon_2]$ and $\sigma_0 > (1-b)[\sigma_1 + \varepsilon_2]$. Thus, in a neighborhood of σ_0, for all $\sigma_1 \in [\sigma_0 - \overline{\varepsilon}, \sigma_0 + \overline{\varepsilon}]$ we can simplify (14) by writing

$$V(l_2|\,\sigma_1) = -\left\{ \begin{array}{c} \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} (\sigma_1 + \varepsilon_2 - l_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } \ l_2 < \sigma_1 - \overline{\varepsilon} \\ \int_{-\overline{\varepsilon}}^{l_2 - \sigma_1} (l_2 - (\sigma_1 + \varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{l_2 - \sigma_1}^{\overline{\varepsilon}} (\sigma_1 + \varepsilon_2 - l_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } l_2 \in [\sigma_1 - \overline{\varepsilon}, \sigma_1 + \overline{\varepsilon}] \\ \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} (l_2 - (\sigma_1 + \varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } \ l_2 > \sigma_1 + \overline{\varepsilon} \end{array} \right. . \tag{21}$$

The next point to note is that l_2 will be less than $\sigma_1 - \overline{\varepsilon}$ if and only if $l_2 - (\sigma_0 - \overline{\varepsilon}) < \varepsilon_1$. Similarly, l_2 will be greater than $\sigma_1 + \overline{\varepsilon}$ if and only if $l_2 - (\sigma_0 + \overline{\varepsilon}) > \varepsilon_1$. We can use these observations along with (21) to write

$$\begin{aligned}
& \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,\sigma_0 + \varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}} \\
= & -\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\overline{\varepsilon})} \left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} (l_2 - (\sigma_0 + \varepsilon_1 + \varepsilon_2))\,\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}} \\
& -\int_{l_2-(\sigma_0+\overline{\varepsilon})}^{l_2-(\sigma_0-\overline{\varepsilon})} \left[\begin{array}{c} \int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\varepsilon_1)} (l_2 - (\sigma_0 + \varepsilon_1 + \varepsilon_2))\,\frac{d\varepsilon_2}{2\overline{\varepsilon}} \\ + \int_{l_2-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}} ((\sigma_0 + \varepsilon_1 + \varepsilon_2) - l_2)\frac{d\varepsilon_2}{2\overline{\varepsilon}} \end{array}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}} \\
& -\int_{l_2-(\sigma_0-\overline{\varepsilon})}^{\overline{\varepsilon}} \left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} ((\sigma_0 + \varepsilon_1 + \varepsilon_2) - l_2)\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}.
\end{aligned}$$

It follows that in a neighborhood of σ_0, we have that

$$\begin{aligned}
& \frac{d\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,\sigma_0 + \varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}}{dl_2} \\
= & -\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\overline{\varepsilon})} \frac{d\varepsilon_1}{2\overline{\varepsilon}} + \int_{l_2-(\sigma_0-\overline{\varepsilon})}^{\overline{\varepsilon}} \frac{d\varepsilon_1}{2\overline{\varepsilon}} - \int_{l_2-(\sigma_0+\overline{\varepsilon})}^{l_2-(\sigma_0-\overline{\varepsilon})} \left[\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\varepsilon_1)} \frac{d\varepsilon_2}{2\overline{\varepsilon}} - \int_{l_2-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}} \frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}.
\end{aligned}$$

It is the case that

$$\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\varepsilon_1)} \frac{d\varepsilon_2}{2\overline{\varepsilon}} - \int_{l_2-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}} \frac{d\varepsilon_2}{2\overline{\varepsilon}} = \frac{1}{\overline{\varepsilon}}\left[l_2 - (\sigma_0 + \varepsilon_1)\right],$$

and thus

$$\begin{aligned}
\int_{l_2-(\sigma_0+\overline{\varepsilon})}^{l_2-(\sigma_0-\overline{\varepsilon})} \left[\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\varepsilon_1)} \frac{d\varepsilon_2}{2\overline{\varepsilon}} - \int_{l_2-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}} \frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}} &= \left[(l_2 - \sigma_0)\,\varepsilon_1 - \frac{\varepsilon_1^2}{2}\right]_{\varepsilon_1 = l_2-(\sigma_0+\overline{\varepsilon})}^{\varepsilon_1 = l_2-(\sigma_0-\overline{\varepsilon})}\left(\frac{1}{2\,(\overline{\varepsilon})^2}\right) \\
&= \left[(l_2 - \sigma_0)\,2\overline{\varepsilon} - (l_2 - \sigma_0)\,2\overline{\varepsilon}\right]\left(\frac{1}{2\,(\overline{\varepsilon})^2}\right) = 0.
\end{aligned}$$

This implies that

$$\frac{d\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,\sigma_0 + \varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}}{dl_2} = -\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\overline{\varepsilon})} \frac{d\varepsilon_1}{2\overline{\varepsilon}} + \int_{l_2-(\sigma_0-\overline{\varepsilon})}^{\overline{\varepsilon}} \frac{d\varepsilon_1}{2\overline{\varepsilon}} = \frac{\sigma_0 - l_2}{\overline{\varepsilon}}.$$

The voter's period 2 expected welfare is therefore increasing in l_2 for l_2 smaller than σ_0 and decreasing for l_2 larger than σ_0. This implies that $\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,\sigma_0 + \varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}$ achieves a local maximum at σ_0.

It remains to show that a higher level of second period welfare cannot be obtained from a limit significantly above or below σ_0. We deal first with the case in which l_2 is significantly below

σ_0. Note that the argument that we have just outlined applies to limits that have the property that for all $\sigma_1 \in [\sigma_0 - \overline{\varepsilon}, \sigma_0 + \overline{\varepsilon}]$ and $\varepsilon_2 \in [-\overline{\varepsilon}, \overline{\varepsilon}]$ it is the case that $l_2 < (b+1)[\sigma_1 + \varepsilon_2]$ and $l_2 > (1-b)[\sigma_1 + \varepsilon_2]$. While the first inequality is true for all l_2 smaller than σ_0, the second may not be. In particular, if $l_2 \leq (1-b)(\sigma_1 + \overline{\varepsilon})$, we need to modify the argument.

Note first that given Assumption 1, if $l_2 \leq (1-b)(\sigma_1 + \overline{\varepsilon})$ then l_2 is less than $\sigma_1 - \overline{\varepsilon}$. It follows from this observation and (14) that welfare in this case is

$$V(l_2 | \sigma_1) = -\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} [\min\{2(\sigma_1 + \varepsilon) - l_2, (1+b)(\sigma_1 + \varepsilon)\} - (\sigma_1 + \varepsilon)] \frac{d\varepsilon}{2\overline{\varepsilon}}.$$

Thus, over this range of limits, period 2 expected welfare is

$$\begin{aligned} & \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2 | \sigma_0 + \varepsilon_1) \frac{d\varepsilon_1}{2\overline{\varepsilon}} \\ = \quad & -\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} (\min\{2(\sigma_0 + \varepsilon_1 + \varepsilon_2) - l_2, (1+b)(\sigma_0 + \varepsilon_1 + \varepsilon_2)\} - ((\sigma_0 + \varepsilon_1 + \varepsilon_2))) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \right] \frac{d\varepsilon_1}{2\overline{\varepsilon}} \end{aligned}$$

By inspection, welfare in this case is increasing in the limit. It follows that the optimal limit cannot be such that $l_2 \leq (1-b)(\sigma_1 + \overline{\varepsilon})$. Since the original argument applies for limits for which $l_2 > (1-b)(\sigma_1 + \overline{\varepsilon})$, we can conclude that the optimal period 2 limit cannot lie below σ_0.

We now turn to the case in which l_2 is significantly above σ_0. Again, our earlier argument applies to limits that have the property that for all $\sigma_1 \in [\sigma_0 - \overline{\varepsilon}, \sigma_0 + \overline{\varepsilon}]$ and $\varepsilon_2 \in [-\overline{\varepsilon}, \overline{\varepsilon}]$, it is the case that $l_2 < (b+1)[\sigma_1 + \varepsilon_2]$ and $l_2 > (1-b)[\sigma_1 + \varepsilon_2]$. We know that the second inequality is true for all l_2 larger than σ_0, which means that $\min\{2(\sigma_1 + \varepsilon) - l_2, (1+b)(\sigma_1 + \varepsilon)\}$ is equal to $2(\sigma_1 + \varepsilon) - l_2$. But the first inequality may not be true, which means that it is possible that $\min\{l_2, (1+b)(\sigma_1 + \varepsilon)\}$ equals $(1+b)(\sigma_1 + \varepsilon)$. Thus, we are interested in the range of limits such that

$$l_2 > (1+b)(\sigma_0 - 2\overline{\varepsilon}). \tag{22}$$

For this range of limits, we cannot rule out the possibility that $l_2 > \sigma_1 + \overline{\varepsilon}$. However, Assumption 1 and (22) imply that $l_2 > \sigma_1 - \overline{\varepsilon}$. This means that we can simplify (14) as follows:

$$V(l_2 | \sigma_1) = -\left\{ \begin{array}{c} \int_{-\overline{\varepsilon}}^{l_2 - \sigma_1} (\min\{l_2, (1+b)(\sigma_1 + \varepsilon_2)\} - (\sigma_1 + \varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{l_2 - \sigma_1}^{\overline{\varepsilon}} (\sigma_1 + \varepsilon_2 - l_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \\ \text{if } \ l_2 \in [\sigma_1 - \overline{\varepsilon}, \sigma_1 + \overline{\varepsilon}] \\ \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} (\min\{l_2, (1+b)(\sigma_1 + \varepsilon_2)\} - (\sigma_1 + \varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } \ l_2 > \sigma_1 + \overline{\varepsilon} \end{array} \right.$$

or expressing the intervals in terms of σ_1

$$V(l_2|\,\sigma_1) = \begin{cases} \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \left(\min\{l_2, (1+b)(\sigma_1+\varepsilon_2)\} - (\sigma_1+\varepsilon_2)\right) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } \sigma_1 < l_2 - \overline{\varepsilon} \\ \int_{-\overline{\varepsilon}}^{l_2-\sigma_1} \left(\min\{l_2, (1+b)(\sigma_1+\varepsilon_2)\} - (\sigma_1+\varepsilon_2)\right) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{l_2-\sigma_1}^{\overline{\varepsilon}} (\sigma_1+\varepsilon_2-l_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \\ \text{if } \sigma_1 \in [l_2-\overline{\varepsilon}, l_2+\overline{\varepsilon}] \end{cases}$$

To make progress with this expression, we need to know the value of $\min\{l_2, (1+b)(\sigma_1+\varepsilon_2)\}$. Consider first the range in which $\sigma_1 < l_2 - \overline{\varepsilon}$. Then

$$\min\{l_2, (1+b)(\sigma_1+\varepsilon_2)\} = \begin{cases} (1+b)(\sigma_1+\varepsilon_2) \text{ if } \frac{l_2}{1+b} - \sigma_1 \geq \varepsilon_2 \\ l_2 \text{ if } \frac{l_2}{1+b} - \sigma_1 < \varepsilon_2 \end{cases}.$$

It follows that if $\frac{l_2}{1+b} - \sigma_1 \in [-\overline{\varepsilon}, \overline{\varepsilon}]$, we have

$$V(l_2|\,\sigma_1) = -\left(\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b}-\sigma_1} b(\sigma_1+\varepsilon_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{\frac{l_2}{1+b}-\sigma_1}^{\overline{\varepsilon}} (l_2 - (\sigma_1+\varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}}\right).$$

Note that

$$\frac{l_2}{1+b} - \sigma_1 \in [-\overline{\varepsilon}, \overline{\varepsilon}] \Leftrightarrow \sigma_1 \in [\frac{l_2}{1+b} - \overline{\varepsilon}, \frac{l_2}{1+b} + \overline{\varepsilon}]$$

Moreover, Assumption 1 implies that $\frac{l_2}{1+b} + \overline{\varepsilon} < l_2 - \overline{\varepsilon}$. Thus, if $\sigma_1 \in [\frac{l_2}{1+b} - \overline{\varepsilon}, \frac{l_2}{1+b} + \overline{\varepsilon}]$, we can write

$$V(l_2|\,\sigma_1) = -\left(\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b}-\sigma_1} b(\sigma_1+\varepsilon_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{\frac{l_2}{1+b}-\sigma_1}^{\overline{\varepsilon}} (l_2 - (\sigma_1+\varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}}\right).$$

If $\sigma_1 \in (\frac{l_2}{1+b} + \overline{\varepsilon}, l_2 - \overline{\varepsilon})$ then it is the case that $\frac{l_2}{1+b} - \sigma_1 < -\overline{\varepsilon}$, so that

$$V(l_2|\,\sigma_1) = -\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} (l_2 - (\sigma_1+\varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}}.$$

If $\sigma_1 < \frac{l_2}{1+b} - \overline{\varepsilon}$ then it is the case that $\frac{l_2}{1+b} - \sigma_1 > \overline{\varepsilon}$, and thus

$$V(l_2|\,\sigma_1) = -\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} b(\sigma_1+\varepsilon_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}}.$$

Putting all this together, we conclude that when $\sigma_1 < l_2 - \overline{\varepsilon}$

$$V(l_2|\,\sigma_1) = -\begin{cases} \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} b(\sigma_1+\varepsilon_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } \sigma_1 < \frac{l_2}{1+b} - \overline{\varepsilon} \\ \left(\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b}-\sigma_1} b(\sigma_1+\varepsilon_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{\frac{l_2}{1+b}-\sigma_1}^{\overline{\varepsilon}} (l_2 - (\sigma_1+\varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}}\right) \text{ if } \sigma_1 \in [\frac{l_2}{1+b} - \overline{\varepsilon}, \frac{l_2}{1+b} + \overline{\varepsilon}] \\ \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} (l_2 - (\sigma_1+\varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } \sigma_1 \in (\frac{l_2}{1+b} + \overline{\varepsilon}, l_2 - \overline{\varepsilon}) \end{cases}.$$

Now consider the range in which $\sigma_1 \in [l_2 - \overline{\varepsilon}, l_2 + \overline{\varepsilon}]$. In this range, we have

$$\sigma_1 > \frac{l_2}{1+b} + \overline{\varepsilon} \Leftrightarrow (1+b)(\sigma_1 - \overline{\varepsilon}) > l_2.$$

Thus, we have $\min\{l_2, (1+b)(\sigma_1 + \varepsilon_2)\} = l_2$. It follows that

$$V(l_2 | \sigma_1) = -\left(\int_{-\overline{\varepsilon}}^{l_2 - \sigma_1} (l_2 - (\sigma_1 + \varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{l_2 - \sigma_1}^{\overline{\varepsilon}} (\sigma_1 + \varepsilon_2 - l_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \right).$$

In summary, when $l_2 > \sigma_0$, we have that

$$V(l_2 | \sigma_1) = - \begin{cases} \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} b(\sigma_1 + \varepsilon_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } \sigma_1 < \frac{l_2}{1+b} - \overline{\varepsilon} \\ \int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b} - \sigma_1} b(\sigma_1 + \varepsilon_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{\frac{l_2}{1+b} - \sigma_1}^{\overline{\varepsilon}} (l_2 - (\sigma_1 + \varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } \sigma_1 \in [\frac{l_2}{1+b} - \overline{\varepsilon}, \frac{l_2}{1+b} + \overline{\varepsilon}] \\ \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} (l_2 - (\sigma_1 + \varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } \sigma_1 \in (\frac{l_2}{1+b} + \overline{\varepsilon}, l_2 - \overline{\varepsilon}) \\ \int_{-\overline{\varepsilon}}^{l_2 - \sigma_1} (l_2 - (\sigma_1 + \varepsilon_2)) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{l_2 - \sigma_1}^{\overline{\varepsilon}} (\sigma_1 + \varepsilon_2 - l_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \text{ if } \sigma_1 \in [l_2 - \overline{\varepsilon}, l_2 + \overline{\varepsilon}] \end{cases}. \tag{23}$$

Next observe that

$$\sigma_1 < \frac{l_2}{1+b} - \overline{\varepsilon} \Leftrightarrow \frac{l_2}{1+b} - \sigma_0 - \overline{\varepsilon} > \varepsilon_1, \tag{24}$$

and that

$$\sigma_1 < \frac{l_2}{1+b} + \overline{\varepsilon} \Leftrightarrow \overline{\varepsilon} + \frac{l_2}{1+b} - \sigma_0 > \varepsilon_1. \tag{25}$$

Similarly,

$$\sigma_1 < l_2 - \overline{\varepsilon} \Leftrightarrow l_2 - \sigma_0 - \overline{\varepsilon} > \varepsilon_1, \tag{26}$$

and that

$$\sigma_1 < l_2 + \overline{\varepsilon} \Leftrightarrow \overline{\varepsilon} + l_2 - \sigma_0 > \varepsilon_1. \tag{27}$$

All this leaves us with four different ranges of limits to consider:

Range 1 The first range is

$$\frac{l_2}{1+b} - \sigma_0 - \overline{\varepsilon} \geq \overline{\varepsilon} \Leftrightarrow l_2 \geq (1+b)(\sigma_0 + 2\overline{\varepsilon})$$

In this case, from (24) we have that for all ε_1 $\sigma_0 + \varepsilon_1 < \frac{l_2}{1+b} - \overline{\varepsilon}$. Thus from (23), we have that

$$\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2 | \sigma_0 + \varepsilon_1) \frac{d\varepsilon_1}{2\overline{\varepsilon}} = -\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} b(\sigma_0 + \varepsilon_1 + \varepsilon_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \right] \frac{d\varepsilon_1}{2\overline{\varepsilon}}.$$

In this range, the voter's expected period 2 welfare is independent of the limit, so that

$$\frac{d\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}}{dl_2}=0.$$

Range 2 The next range is

$$\begin{aligned}\frac{l_2}{1+b}-\sigma_0-\overline{\varepsilon} &< \overline{\varepsilon}\leq\overline{\varepsilon}+\frac{l_2}{1+b}-\sigma_0\\ &\Leftrightarrow l_2\in[(1+b)\sigma_0,(1+b)\left(\sigma_0+2\overline{\varepsilon}\right))\end{aligned}$$

In this case, from (25) we have that for all ε_1 $\sigma_0+\varepsilon_1<\frac{l_2}{1+b}+\overline{\varepsilon}$. For realizations of ε_1 less than $\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})$ we have that $\sigma_0+\varepsilon_1<\frac{l_2}{1+b}-\overline{\varepsilon}$, while for realizations in excess of this threshold, we have $\sigma_0+\varepsilon_1\in[\frac{l_2}{1+b}-\overline{\varepsilon},\frac{l_2}{1+b}+\overline{\varepsilon}]$. Thus from (23), we have that

$$\begin{aligned}&\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}=-\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})}\left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} b\left(\sigma_0+\varepsilon_1+\varepsilon_2\right)\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}\\ &-\int_{\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})}^{\overline{\varepsilon}}\left[\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b}-(\sigma_0+\varepsilon_1)} b\left(\sigma_0+\varepsilon_1+\varepsilon_2\right)\frac{d\varepsilon_2}{2\overline{\varepsilon}}+\int_{\frac{l_2}{1+b}-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}}\left(l_2-\left(\sigma_0+\varepsilon_1+\varepsilon_2\right)\right)\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}.\end{aligned}$$

In this case

$$\frac{d\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,(1+g)\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}}{dl_2}=-\int_{\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})}^{\overline{\varepsilon}}\left[\int_{\frac{l_2}{1+b}-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}}\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}<0.$$

Range 3 The next range is that

$$\overline{\varepsilon}+\frac{l_2}{1+b}-\sigma_0<\overline{\varepsilon}\leq l_2-\sigma_0-\overline{\varepsilon}\Leftrightarrow l_2\in[\sigma_0+2\overline{\varepsilon},(1+b)\sigma_0).$$

In this case, from (26) we have that for all ε_1 $\sigma_0+\varepsilon_1<l_2-\overline{\varepsilon}$. There are two possibilities:

Possibility i) $l_2<(1+b)\left(\sigma_0-2\overline{\varepsilon}\right)$ in which case for all ε_1 $\sigma_0+\varepsilon_1\in(\frac{l_2}{1+b}+\overline{\varepsilon},l_2-\overline{\varepsilon})$, and thus from (23), we have that

$$\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}=-\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}}\left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}}\left(l_2-\left(\sigma_0+\varepsilon_1+\varepsilon_2\right)\right)\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}.$$

It follows in this case that

$$\frac{d\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}}{dl_2}=-\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}}\frac{d\varepsilon_1}{2\overline{\varepsilon}}<0.$$

Possibility ii) $(1+b)(\sigma_0 - 2\overline{\varepsilon}) < l_2 < (1+b)\sigma_0$ in which case for realizations of ε_1 less than $\frac{l_2}{1+b} - (\sigma_0 - \overline{\varepsilon})$ we have that

$$\sigma_0 + \varepsilon_1 \in [\frac{l_2}{1+b} - \overline{\varepsilon}, \frac{l_2}{1+b} + \overline{\varepsilon}],$$

while for realizations in excess of this threshold we have

$$\sigma_0 + \varepsilon_1 \in (\frac{l_2}{1+b} + \overline{\varepsilon}, l_2 - \overline{\varepsilon}).$$

Thus from (23), we have that

$$\begin{aligned}
&\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2 | \sigma_0 + \varepsilon_1) \frac{d\varepsilon_1}{2\overline{\varepsilon}} \\
= \;& -\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b} - (\sigma_0 - \overline{\varepsilon})} \left[\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b} - (\sigma_0 + \varepsilon_1)} b\left(\sigma_0 + \varepsilon_1 + \varepsilon_2\right) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{\frac{l_2}{1+b} - (\sigma_0 + \varepsilon_1)}^{\overline{\varepsilon}} \left(l_2 - (\sigma_0 + \varepsilon_1 + \varepsilon_2)\right) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \right] \frac{d\varepsilon_1}{2\overline{\varepsilon}} \\
& -\int_{\frac{l_2}{1+b} - (\sigma_0 - \overline{\varepsilon})}^{\overline{\varepsilon}} \left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \left(l_2 - (\sigma_0 + \varepsilon_1 + \varepsilon_2)\right) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \right] \frac{d\varepsilon_1}{2\overline{\varepsilon}}.
\end{aligned}$$

It follows in this case that

$$\frac{d \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2 | \sigma_0 + \varepsilon_1) \frac{d\varepsilon_1}{2\overline{\varepsilon}}}{dl_2} = -\left(\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b} - (\sigma_0 - \overline{\varepsilon})} \left[\int_{\frac{l_2}{1+b} - (\sigma_0 + \varepsilon_1)}^{\overline{\varepsilon}} \frac{d\varepsilon_2}{2\overline{\varepsilon}} \right] \frac{d\varepsilon_1}{2\overline{\varepsilon}} + \int_{\frac{l_2}{1+b} - (\sigma_0 - \overline{\varepsilon})}^{\overline{\varepsilon}} \frac{d\varepsilon_1}{2\overline{\varepsilon}} \right) < 0.$$

Range 4 The final range is that

$$l_2 - \sigma_0 - \overline{\varepsilon} < \overline{\varepsilon} \le \overline{\varepsilon} + l_2 - \sigma_0 \Leftrightarrow l_2 \in [\sigma_0, \sigma_0 + 2\overline{\varepsilon}).$$

In this case, from (27), we have that for all ε_1 $\sigma_0 + \varepsilon_1 < l_2 + \overline{\varepsilon}$. There are three possibilities:

Possibility i) $l_2 < (1+b)(\sigma_0 - 2\overline{\varepsilon})$ in which case for realizations of ε_1 less than $l_2 - (\sigma_0 + \overline{\varepsilon})$ we have that $\sigma_0 + \varepsilon_1 \in (\frac{l_2}{1+b} + \overline{\varepsilon}, l_2 - \overline{\varepsilon})$, while for realizations in excess of this threshold $\sigma_0 + \varepsilon_1 \in [l_2 - \overline{\varepsilon}, l_2 + \overline{\varepsilon}]$. Thus, from (23), we have that

$$\begin{aligned}
&\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2 | \sigma_0 + \varepsilon_1) \frac{d\varepsilon_1}{2\overline{\varepsilon}} = -\int_{-\overline{\varepsilon}}^{l_2 - (\sigma_0 + \overline{\varepsilon})} \left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \left(l_2 - (\sigma_0 + \varepsilon_1 + \varepsilon_2)\right) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \right] \frac{d\varepsilon_1}{2\overline{\varepsilon}} \\
& -\int_{l_2 - (\sigma_0 + \overline{\varepsilon})}^{\overline{\varepsilon}} \left[\int_{-\overline{\varepsilon}}^{l_2 - (\sigma_0 + \varepsilon_1)} \left(l_2 - (\sigma_0 + \varepsilon_1 + \varepsilon_2)\right) \frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{l_2 - (\sigma_0 + \varepsilon_1)}^{\overline{\varepsilon}} (\sigma_0 + \varepsilon_1 + \varepsilon_2 - l_2) \frac{d\varepsilon_2}{2\overline{\varepsilon}} \right] \frac{d\varepsilon_1}{2\overline{\varepsilon}}.
\end{aligned}$$

It follows that

$$\frac{d \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2 | \sigma_0 + \varepsilon_1) \frac{d\varepsilon_1}{2\overline{\varepsilon}}}{dl_2} = -\left(\int_{-\overline{\varepsilon}}^{l_2 - (\sigma_0 + \overline{\varepsilon})} \frac{d\varepsilon_1}{2\overline{\varepsilon}} + \int_{l_2 - (\sigma_0 + \overline{\varepsilon})}^{\overline{\varepsilon}} \left[\int_{-\overline{\varepsilon}}^{l_2 - (\sigma_0 + \varepsilon_1)} \frac{d\varepsilon_2}{2\overline{\varepsilon}} - \int_{l_2 - (\sigma_0 + \varepsilon_1)}^{\overline{\varepsilon}} \frac{d\varepsilon_2}{2\overline{\varepsilon}} \right] \frac{d\varepsilon_1}{2\overline{\varepsilon}} \right).$$

We claim that

$$\int_{l_2-(\sigma_0+\overline{\varepsilon})}^{\overline{\varepsilon}}\left[\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\varepsilon_1)}\frac{d\varepsilon_2}{2\overline{\varepsilon}}-\int_{l_2-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}}\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}\geq 0.$$

To see this note first that

$$\begin{aligned}\int_{l_2-(\sigma_0+\overline{\varepsilon})}^{\overline{\varepsilon}}\left[\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\varepsilon_1)}\frac{d\varepsilon_2}{2\overline{\varepsilon}}-\int_{l_2-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}}\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}} &= \left[\frac{(l_2-\sigma_0)\,\varepsilon_1-\frac{\varepsilon_1^2}{2}}{\overline{\varepsilon}}\right]_{\varepsilon_1=l_2-(\sigma_0+\overline{\varepsilon})}^{\varepsilon_1=\overline{\varepsilon}}\\ &= \frac{\overline{\varepsilon}\,(l_2-\sigma_0)-\frac{\overline{\varepsilon}^2}{2}-\frac{(l_2-\sigma_0)^2}{2}+\frac{\overline{\varepsilon}^2}{2}}{\overline{\varepsilon}}\\ &= \frac{(l_2-\sigma_0)\left(\overline{\varepsilon}-\frac{(l_2-\sigma_0)}{2}\right)}{\overline{\varepsilon}}\geq 0\end{aligned}$$

where the inequality follows from the fact that $l_2\leq\sigma_0+2\overline{\varepsilon}$. It follows that

$$\frac{d\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}}V(\,l_2|\,\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}}{dl_2}<0.$$

Possibility ii) $(1+b)\,(\sigma_0-2\overline{\varepsilon})<l_2<(1+b)\sigma_0$ in which case for realizations of ε_1 less than $\frac{l_2}{1+b}-(\sigma_0-\overline{\varepsilon})$ we have $\sigma_0+\varepsilon_1\in[\frac{l_2}{1+b}-\overline{\varepsilon},\frac{l_2}{1+b}+\overline{\varepsilon}]$, for realizations between $\frac{l_2}{1+b}-(\sigma_0-\overline{\varepsilon})$ and $l_2-(\sigma_0+\overline{\varepsilon})$ we have that $\sigma_0+\varepsilon_1\in(\frac{l_2}{1+b}+\overline{\varepsilon},l_2-\overline{\varepsilon})$, while for realizations in excess of this threshold $\sigma_0+\varepsilon_1\in[l_2-\overline{\varepsilon},l_2+\overline{\varepsilon}]$. Thus, from (23), we have that

$$\begin{aligned}\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}}V(\,l_2|\,\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}} &= -\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})}\left[\begin{array}{c}\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b}-(\sigma_0+\varepsilon_1)}b\,(\sigma_0+\varepsilon_1+\varepsilon_2)\,\frac{d\varepsilon_2}{2\overline{\varepsilon}}\\ +\int_{\frac{l_2}{1+b}-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}}(l_2-(\sigma_0+\varepsilon_1+\varepsilon_2))\,\frac{d\varepsilon_2}{2\overline{\varepsilon}}\end{array}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}\\ &-\int_{\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})}^{\frac{l_2}{1+b}-(\sigma_0-\overline{\varepsilon})}\left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}}(l_2-(\sigma_0+\varepsilon_1+\varepsilon_2))\,\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}\\ &-\int_{\frac{l_2}{1+b}-(\sigma_0-\overline{\varepsilon})}^{\overline{\varepsilon}}\left[\begin{array}{c}\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\varepsilon_1)}(l_2-(\sigma_0+\varepsilon_1+\varepsilon_2))\,\frac{d\varepsilon_2}{2\overline{\varepsilon}}\\ +\int_{l_2-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}}((\sigma_0+\varepsilon_1+\varepsilon_2)-l_2)\frac{d\varepsilon_2}{2\overline{\varepsilon}}\end{array}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}.\end{aligned}$$

It follows that

$$\begin{aligned}&\frac{d\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}}V(\,l_2|\,\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}}{dl_2}\\ =\;&-\left[\begin{array}{c}\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})}\left(\int_{\frac{l_2}{1+b}-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}}\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right)\frac{d\varepsilon_1}{2\overline{\varepsilon}}\\ +\left(\int_{\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})}^{\frac{l_2}{1+b}-(\sigma_0-\overline{\varepsilon})}\frac{d\varepsilon_1}{2\overline{\varepsilon}}+\int_{\frac{l_2}{1+b}-(\sigma_0-\overline{\varepsilon})}^{\overline{\varepsilon}}\left[\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\varepsilon_1)}\frac{d\varepsilon_2}{2\overline{\varepsilon}}-\int_{l_2-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}}\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}\right)\end{array}\right].\end{aligned}$$

Given our analysis of possibility i), it is clear that this is negative.

Possibility iii) $l_2 > (1+b)\sigma_0$ in which case for realizations of ε_1 less than $\frac{l_2}{1+b} - (\sigma_0 + \overline{\varepsilon})$ we have $\sigma_0 + \varepsilon_1 < \frac{l_2}{1+b} - \overline{\varepsilon}$, for realizations of ε_1 between $\frac{l_2}{1+b} - (\sigma_0 + \overline{\varepsilon})$ and $\frac{l_2}{1+b} - (\sigma_0 - \overline{\varepsilon})$ we have $\sigma_0 + \varepsilon_1 \in [\frac{l_2}{1+b} - \overline{\varepsilon}, \frac{l_2}{1+b} + \overline{\varepsilon}]$, for realizations between $\frac{l_2}{1+b} - (\sigma_0 - \overline{\varepsilon})$ and $l_2 - (\sigma_0 + \overline{\varepsilon})$ we have that $\sigma_0 + \varepsilon_1 \in (\frac{l_2}{1+b} + \overline{\varepsilon}, l_2 - \overline{\varepsilon})$, while for realizations in excess of this threshold $\sigma_0 + \varepsilon_1 \in [l_2 - \overline{\varepsilon}, l_2 + \overline{\varepsilon}]$. Thus, from (23), we have that

$$
\begin{aligned}
\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}} &= -\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})} \left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} b\left(\sigma_0+\varepsilon_1+\varepsilon_2\right)\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}} \\
&- \int_{\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})}^{\frac{l_2}{1+b}-(\sigma_0-\overline{\varepsilon})} \left[\int_{-\overline{\varepsilon}}^{\frac{l_2}{1+b}-(\sigma_0+\varepsilon_1)} b\left(\sigma_0+\varepsilon_1+\varepsilon_2\right)\frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{\frac{l_2}{1+b}-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}} \left(l_2-(\sigma_0+\varepsilon_1+\varepsilon_2)\right)\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}} \\
&- \int_{\frac{l_2}{1+b}-(\sigma_0-\overline{\varepsilon})}^{l_2-(\sigma_0+\overline{\varepsilon})} \left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \left(l_2-(\sigma_0+\varepsilon_1+\varepsilon_2)\right)\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}} \\
&- \int_{l_2-(\sigma_0+\overline{\varepsilon})}^{\overline{\varepsilon}} \left[\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\varepsilon_1)} \left(l_2-(\sigma_0+\varepsilon_1+\varepsilon_2)\right)\frac{d\varepsilon_2}{2\overline{\varepsilon}} + \int_{l_2-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}} \left((\sigma_0+\varepsilon_1+\varepsilon_2)-l_2\right)\frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}
\end{aligned}
$$

It follows that

$$
\begin{aligned}
&\frac{d\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(l_2|\,\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}}{dl_2} \\
= \quad & -\left[\begin{array}{c} \int_{\frac{l_2}{1+b}-(\sigma_0+\overline{\varepsilon})}^{\frac{l_2}{1+b}-(\sigma_0-\overline{\varepsilon})} \left(\int_{\frac{l_2}{1+b}-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}} \frac{d\varepsilon_2}{2\overline{\varepsilon}}\right)\frac{d\varepsilon_1}{2\overline{\varepsilon}} \\ +\left(\int_{\frac{l_2}{1+b}-(\sigma_0-\overline{\varepsilon})}^{l_2-(\sigma_0+\overline{\varepsilon})} \frac{d\varepsilon_1}{2\overline{\varepsilon}} + \int_{l_2-(\sigma_0+\overline{\varepsilon})}^{\overline{\varepsilon}} \left[\int_{-\overline{\varepsilon}}^{l_2-(\sigma_0+\varepsilon_1)} \frac{d\varepsilon_2}{2\overline{\varepsilon}} - \int_{l_2-(\sigma_0+\varepsilon_1)}^{\overline{\varepsilon}} \frac{d\varepsilon_2}{2\overline{\varepsilon}}\right]\frac{d\varepsilon_1}{2\overline{\varepsilon}}\right) \end{array}\right]
\end{aligned}
$$

Given our analysis of possibility i), it is clear that this is negative. ■

6.6 Proof of Proposition 5

The voter's expected welfare under the sequence $\{l_1, l_2\} = \{\sigma_0, \sigma_0\}$ is given by

$$\pi\left[y_1 V(\sigma_0|\,\sigma_0) + \beta y_2 \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(\sigma_0|\,\sigma_0+\varepsilon_1)\frac{d\varepsilon_1}{2\overline{\varepsilon}}\right], \tag{28}$$

where the function $V(l|\,\sigma)$ is as defined in (14). So we need to show that the sequence of spending-contingent limits $\{l_1, l_2\} = \{\sigma_0, s_1/y_1\}$ generates the voter a strictly higher expected payoff than (28).

Consider what happens under the sequence of spending-contingent limits $\{l_1, l_2\} = \{\sigma_0, s_1/y_1\}$. We begin in period 2. Suppose that the period 1 shock was ε_1. There are two possibilities: either the period 1 politician was biased or he was unbiased. In the latter case, $s_1 = (\sigma_0 + \varepsilon_1)\, y_1$ and thus $l_2 = \sigma_0 + \varepsilon_1$. The voter's period 2 expected payoff is therefore $\pi y_2 V(\sigma_0 + \varepsilon_1|\,\sigma_0 + \varepsilon_1)$. In the former case, let $s_1(\varepsilon_1)$ denote the policy choice made by the period 1 politician. Then, the voter's period 2 expected payoff is $\pi y_2 V(\frac{s_1(\varepsilon_1)}{y_1}\Big|\, \sigma_0 + \varepsilon_1)$.

Now consider the policy choice $s_1(\varepsilon_1)$ made by the biased politician in period 1. There are two possibilities: the first is that the policy $s_1(\varepsilon_1)$ respects the limit; i.e., $s_1(\varepsilon_1) \leq \sigma_0 y_1$. In this case, we claim that it must be the case that $s_1(\varepsilon_1)$ equals $\sigma_0 y_1$. It suffices to show that for all ε_1 it must be the case that the biased politician prefers a higher level of policy than $\sigma_0 y_1$; that is, $(1+b)(\sigma_0 + \varepsilon_1) \geq \sigma_0$. This is true if and only if $(1+b)(\sigma_0 - \overline{\varepsilon}) \geq \sigma_0$, which follows from Assumption 1. The fact that $s_1(\varepsilon_1)$ equals $\sigma_0 y_1$ in this case means that the voter's period 2 expected payoff is $\pi y_2 V(\sigma_0 |\, \sigma_0 + \varepsilon_1)$. His two period payoff is therefore

$$-y_1 \left|\sigma_0 - (\sigma_0 + \varepsilon_1)\right| + \beta \pi y_2 V(\sigma_0 |\, \sigma_0 + \varepsilon_1). \tag{29}$$

The second possibility is that the policy $s_1(\varepsilon_1)$ exceeds the limit; i.e., $s_1(\varepsilon_1) > \sigma_0 y$. In this case, given that the voter knows that the politician will choose policy $\sigma_0 y_1$ if he respects the limit, by revealed preference we must have

$$\begin{aligned} & -y_1 \left|s_1(\varepsilon_1) - (\sigma_0 + \varepsilon_1)\right| + \beta \pi y_2 V(\frac{s_1(\varepsilon_1)}{y_1}\Big|\, \sigma_0 + \varepsilon_1) \\ \geq \quad & -y_1 \left|\sigma_0 - (\sigma_0 + \varepsilon_1)\right| + \beta \pi y_2 V(\sigma_0 |\, \sigma_0 + \varepsilon_1). \end{aligned}$$

Otherwise the voter would not have approved the proposal $s_1(\varepsilon_1)$. The voter's two period payoff therefore is at least as large as (29).

Taking expectations over ε_1 and the probability that the period 1 politician is biased, we conclude that the voter's expected payoff under the sequence of spending-contingent limits $\{l_1, l_2\} = \{\sigma_0, s_1/y_1\}$ is at least as large as

$$\begin{aligned} & \pi \left[\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \left[-y_1 \left|\sigma_0 - (\sigma_0 + \varepsilon_1)\right| + \beta \pi y_2 V(\sigma_0 |\, \sigma_0 + \varepsilon_1)\right] \frac{d\varepsilon_1}{2\overline{\varepsilon}} \right] \\ & + (1-\pi) \left[\beta \pi y_2 \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(\sigma_0 + \varepsilon_1 |\, \sigma_0 + \varepsilon_1) \frac{d\varepsilon_1}{2\overline{\varepsilon}} \right]. \end{aligned}$$

Now observe that

$$-\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \left[\left|\sigma_0 - (\sigma_0 + \varepsilon_1)\right|\right] \frac{d\varepsilon_1}{2\overline{\varepsilon}} = V(\sigma_0 |\, \sigma_0)$$

where the function $V(l|\, \sigma)$ is as defined in (14). To see this, note that by definition

$$V(\sigma_0 |\, \sigma_0) = -\left\{ \int_{-\overline{\varepsilon}}^{0} (\sigma_0 - (\sigma_0 + \varepsilon)) \frac{d\varepsilon}{2\overline{\varepsilon}} + \int_0^{\overline{\varepsilon}} (\min\{\sigma_0 + 2\varepsilon, (1+b)(\sigma_0 + \varepsilon)\} - (\sigma_0 + \varepsilon)) \frac{d\varepsilon}{2\overline{\varepsilon}} \right\}$$

But Assumption 1 implies that $\sigma_0 + 2\varepsilon < (1+b)(\sigma_0 + \varepsilon)$.

It follows that the voter's expected payoff under the sequence of spending-contingent limits $\{l_1, l_2\} = \{\sigma_0, s_1/y_1\}$ is at least as large as

$$\pi y_1 V(\sigma_0 | \sigma_0) + \pi^2 \beta y_2 \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(\sigma_0 | \sigma_0 + \varepsilon_1) \frac{d\varepsilon_1}{2\overline{\varepsilon}} + (1-\pi)\pi\beta y_2 \int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} V(\sigma_0 + \varepsilon_1 | \sigma_0 + \varepsilon_1) \frac{d\varepsilon_1}{2\overline{\varepsilon}}$$

This strictly exceeds the voter's expected welfare under the sequence $\{l_1, l_2\} = \{\sigma_0, \sigma_0\}$ as given by (28). $\blacksquare$